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Zeno effect preventing Rabi transitions onto an unstable energy level

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Abstract

We consider a driven 2-level system with one level showing spontaneous decay to an otherwise uncoupled third level. Rabi transitions to the unstable level are strongly damped. This simple configuration can be used to demonstrate and to explore the quantum Zeno effect leading to a freezing of the system to the initial level. A comparison with repeated projection measurements is given. A treatment within a phenomenological theory of continuous measurements is sketched. The system visualizes the important role of null measurements (negative result measurements) and may serve as a good example for a truly continuous measurement. © 1999 Published by Elsevier Science B.V. All rights reserved.

1. Introduction

The quantum Zeno effect [1–4] is one of the specific quantum effects which have gained much interest in particular in connection with the quantum theory of measurement. It goes back to successive or truly continuous measurements of the system and depends on the strength of the measurement. Generally speaking, it is the slowing down of induced quantum transitions as a result of repeated measurements of the system. In the limit of a continuous measurement, transitions of the system are inhibited. The system is frozen on a level.

Characteristic for the Zeno effect is that it is caused by measurement. A prevention of a transition must not necessarily be a manifestation of the Zeno effect. It may also happen that a Zeno effect is accompanied by other influences. In the following we discuss a new demonstration of a pure Zeno effect.

A well known particular example of the Zeno effect is the slowing down of Rabi transitions between two energy levels $|1\rangle$ and $|2\rangle$ of a 2-level system, which are induced by a resonant driving laser field, as the result of frequently repeated energy measurements. In the well-known experiment of Itano et al. [4] the repeated energy measurement is realized for ions with the help of periodical short pulses of another laser field inducing optical transitions from level $|1\rangle$ onto a third level $|3\rangle$ with subsequent spontaneous decay back to the initial level $|1\rangle$ (V-configuration). The photon emitted by spontaneous emission is monitored. If the ion is on level $|1\rangle$, it will emit photons. This does not happen if it is on

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level $|2\rangle$. In this sense the fluorescence photons contain information about the state of the ion. The $|1\rangle \rightarrow |3\rangle$ transition together with the emission of the fluorescence photon represents a non-destructive projection measurement of level $|1\rangle$. As a result the transition $|1\rangle \rightarrow |2\rangle$ is hindered. This freezing on a level is a pure demonstration of quantum Zeno effect caused by repeated almost instantaneous projection measurements (comp. [5], where also a more complete list of the literature can be found).

The aim of this letter is to show that the Zeno effect may be demonstrated in a much simpler way for a driven 2-level system with one level showing spontaneous decay to an otherwise uncoupled third level. At the same time this system may serve as a good example for a truly continuous quantum measurement. In addition it visualizes the important role of null measurements.

We consider the very simple quantum system presented in Fig. 1a. It is a 2-level system with levels (energy eigenstates) $|1\rangle$ and $|2\rangle$ which is subject to the influence of a resonant driving field V generating Rabi oscillations between $|1\rangle$ and $|2\rangle$. Initially the system is on level $|1\rangle$ and the driving field induces a transition to level $|2\rangle$. However level $|2\rangle$ is assumed to be unstable. It decays rapidly by spontaneous decay with relaxation rate γ to a third level $|3\rangle$ with emission of a photon. Accordingly, if the system reaches level $|2\rangle$ it very quickly transits further to level $|3\rangle$. Therefore, if no photon is emitted, this is a sign that the system stays perpetually on level $|1\rangle$. Consequently we may consider this setup as a realization of a continuous measurement of the energy level $|1\rangle$ in a passive way leading to null-results (in contrast to the non-continuous and active scheme realized by Itano et al. [4]).

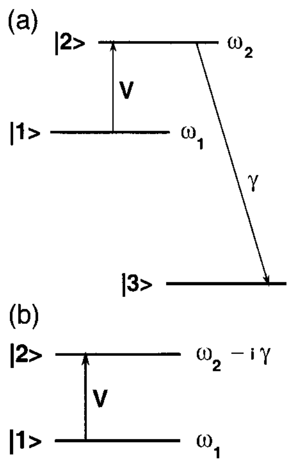

Fig. 1. 3-level system with initial state $|1\rangle$ and Rabi transition to state $|2\rangle$ from where it only may decay to level $|3\rangle$ accompanied by the emission of a photon (a). The equivalent 2-level system has a complex energy of the unstable level $|2\rangle$ (b).

What type of dynamical behavior is to be expected? Naively one could imagine that the system is transferred with the first Rabi pulse to level $|2\rangle$ from where it quickly decays to level $|3\rangle$. The contrary turns out to be the case. The transition $|1\rangle \rightarrow |2\rangle$ is slowed down and the effectivity for this freezing on the initial level $|1\rangle$ actually increases with the effectivity of the spontaneous decay of level $|2\rangle$, i.e. with increasing relaxation rate γ. This may seem to be counterintuitive or paradoxical. But in fact it is no surprise, because in a situation in which a continuous quantum measurement is realized due to the coupling to the environment, the quantum Zeno effect is to be expected. We will show that this behavior of the system can indeed very easily be understood as another experimentally accessible example of a pure manifestation of the Zeno effect. That this effect is in our case based only i) on null-measurements and ii) on genuinely continuous measurements, makes it even more interesting from the conceptional point of view.

We start in Section 2 with a quantum-optical description of the microphysical dynamics underlying this Zeno effect. We compare it in Section 3 with the phenomenological approach based on repeated measurements. In Section 4 we give a treatment within a phenomenological theory of continuous quantum measurements.

2. Quantum-optical treatment of the system

We are dealing with a 3-level system which is under the influence of a driving field and in interaction with the electromagnetic vacuum causing the spontaneous emission of a photon. An adequate treatment of the complete underlying Schrödinger dynamics of the open system is rather involved. Let us therefore introduce already on the level of the microphysical quantum-optical treatment some phenomenological elements. We study instead of the 3-level system an equivalent 2-level system consisting only out of the levels $|1\rangle$ and $|2\rangle$ with energies $\hbar\omega_1$ and $\hbar\omega_2$, and represent the instability of level $|2\rangle$ by an imaginary term in the energy of this level (Fig. 1 b).

We write the Hamiltonian of the system as $H = H_\gamma + V$ with

$$H_\gamma|1\rangle = \hbar\omega_1|1\rangle, \quad H_\gamma|2\rangle = \hbar(\omega_2 - i\gamma)|2\rangle,$$
$$\langle 1|V|2\rangle = \langle 2|V|1\rangle^* = \hbar\Omega\, e^{i(\omega_2-\omega_1)t}. \tag{1}$$

Let us introduce amplitudes $a_1(t)$, $a_2(t)$ as follows:

$$|\psi\rangle = a_1(t)\, e^{-i\omega_1 t}|1\rangle + a_2(t)\, e^{-i\omega_2 t}|2\rangle. \tag{2}$$

The dynamics

$$|\dot{\psi}\rangle = -\frac{i}{\hbar}(H_\gamma + V)|\psi\rangle \tag{3}$$

then result in the following differential equations for the amplitudes

$$\dot{a}_1 = -i\Omega a_2, \quad \dot{a}_2 = -i\Omega a_1 - \gamma a_2. \tag{4}$$

The solution is

$$a_1(t) = e^{-\gamma/2t}\left[a_1(0)\cosh\Omega_\gamma t + \frac{1}{\Omega_\gamma}\left(\frac{\gamma}{2}a_1(0) - i\Omega a_2(0)\right)\sinh\Omega_\gamma t\right],$$
$$a_2(t) = e^{-\gamma/2t}\left[a_2(0)\cosh\Omega_\gamma t - \frac{1}{\Omega_\gamma}\left(i\Omega a_1(0) + \frac{\gamma}{2}a_2(0)\right)\sinh\Omega_\gamma t\right], \tag{5}$$

where we have introduced

$$\Omega_\gamma = \sqrt{(\gamma^2/4) - \Omega^2}. \tag{6}$$

Under the condition that the system is initially ($t=0$) on level $|1\rangle$, we find

$$a_1(t) = e^{-\gamma/2t}\left(\cosh\Omega_\gamma t + \frac{\gamma}{2\Omega_\gamma}\sinh\Omega_\gamma t\right),$$
$$a_2(t) = e^{-\gamma/2t}\left(-i\frac{\Omega}{\Omega_\gamma}\sinh\Omega_\gamma t\right). \tag{7}$$

Depending on the relative strength of the two parameters γ and Ω one can discriminate different regimes of behavior. If the influence of the driving field is strong as compared to the strength of the spontaneous decay, i.e. for $\Omega \gg \gamma$, we are in the Rabi regime, where the behavior of the system shows the usual Rabi oscillations modified by some damping:

$$a_1(t) = e^{-\gamma/2t}\cos\Omega t, \quad a_2(t) = -ie^{-\gamma/2t}\sin\Omega t. \tag{8}$$

We see that the system, being initially on level 1, will be on level 2 after the Rabi period $T_R = \pi/2\Omega$.

For our purpose it is the opposite regime with strong coupling $\gamma \gg \Omega$ to the environment which is of interest. In this case the probability of permanent survival $P_{ps}(T)$ of the system on level $|1\rangle$ during sufficiently long time T turns out to be:

$$P_{ps}(T) = |a_1(T)|^2 = e^{-2\Omega^2 T/\gamma}. \tag{9}$$

Because of $\gamma \gg \Omega$ we find damping of the Rabi oscillations. In the limit $\Omega/\gamma \to 0$ we have $P_{ps}(T) \to 1$ for any fixed T and the system is frozen on level $|1\rangle$. This is a manifestation of the Zeno effect.

We recall that the Zeno effect is the phenomenon that continuous measurement or many consecutive quantum measurements lead to damping and in a limit to freezing of the evolution of the measured system. In our case we have a continuous measurement because of the coupling to the environment via spontaneous decay. And we have just shown that it results in freezing of the evolution of the system.

It is a characteristic trait of our system that it is not closed, even when no photon is emitted. In fact this enduring failure to observe a photon in the interval $[0,T]$ represents the continuum version of a series of nondetection measurements or negative result measurements. We called them null measurements. A null measurement influences the system in just the same way as the usual positive result measurement. It would be therefore misleading to call it an 'interaction free measurement' [6,7]. May be it should be better called an 'energy exchange free measurement' [8]. Our setup may serve as a simple example where the concept of null measurements leading to wave function collapses which result in level freezing can successfully be applied. Other types of null measurements which are performed on the neutron spin [9] or on the photon polarization [10] are discussed in the literature.

Finally we add that a full and detailed quantum-optical treatment, which is not based on a complex energy as in Eq. (1) leads to the same result. It is therefore possible to obtain for our system the quantum Zeno effect within a fully microphysical approach. Note that it was not necessary to refer to a collapse of the wave function. We will now compare these results with the usual phenomenological approach to the Zeno effect, which is based on repeated measurements.

3. Comparison with repeated projection measurements

Let us first consider a situation for which the setup of Itano et al. [4] may be regarded as an experimental realization. We have a system with two levels $|1\rangle$ and $|2\rangle$. We perform a sequence of strong measurements resulting in projections of the state vector (the continuous case will be discussed below). The two corresponding measurement results are R_1 and R_2. If R_i is measured, the system is transferred to level $|i\rangle$. Accordingly the measurement of R_i is equivalent to the information that the system is immediately afterwards on level $|i\rangle$.

Let us assume that the first measurement of a sequence is performed at $t=0$ (thus fixing the initial state) and then repeated N times with time interval τ between the consecutive measurements. τ is the relevant parameter. After the first repetition at time $t=\tau$ the system will still be found on the initial level with probability q or on the other level with probability $p=1-q$. After the time $T=N\tau$ the probability that the system has made k times a transition to the other level and has stayed $N-k$ times on the initial level is $P(k)=p^k q^{N-k}$. The probability that the system will be found finally at $t=T$ on the initial level regardless of what have been the results of the measurements until then is

$$P^Z(T) = \sum_{k\,\mathrm{even}} \binom{N}{k} p^k q^{N-k}. \tag{10}$$

For any fixed T the probability $P^Z(T)$ tends to 1 for $\tau \to 0$ (or $N \to \infty$).

For the experiment of the previous section the Rabi oscillation during the time τ fixes p and q to be

$$p=\sin^2\Omega\tau, \quad q=\cos^2\Omega\tau. \tag{11}$$

To compare with the sequential approach sketched above, we have to take into account that our system is peculiar in the following sense: the first measurement result R_1 is 'no emission of a photon'. This is a null result. We know that the system is on the initial level $|1\rangle$. There is no alternative measurement result R_2 because 'emission of a photon' is not connected with a projection onto $|i\rangle$, but indicates the end of the measurement series. Detection and emission of a photon are identified (see below). We know that all the measurements before have had the null result R_1. The related probability of permanent survival at level $|1\rangle$ is obtained as the first term ($k=0$) of the sum (10):

$$P^Z_{\mathrm{ps}}(T) = \cos^{2N}(\Omega\tau). \tag{12}$$

We turn now to the case of very rapid repetitions with $\tau \ll \Omega^{-1}$. In this limit we may rewrite Eq. (12) without restriction of the time T of permanent survival as

$$P^Z_{\mathrm{ps}}(T) = \exp(-\Omega^2 T\tau). \tag{13}$$

Accordingly the result is that we can reach complete agreement with $P_{\mathrm{ps}}(T)$ of Eq. (9) by identifying

$$\tau = \frac{2}{\gamma}. \tag{14}$$

The repetition time τ must be chosen as the inverse of half of the relaxation rate. The quantum-optical treatment of Section 2 uniquely fixes in our comparison the otherwise unspecified parameter τ. The condition $T \gg \gamma^{-1}$ of Section 2 corresponds to the evident demand $T \gg \tau$. Perfect level freezing is obtained in the limit $\tau \propto \gamma^{-1} \to 0$.

It is interesting to see that because of Eq. (14) the repeated collapse to the ground state happens after time intervals τ, which are completely fixed by the atomic lifetime. No reference to a photon detector is necessary. This demonstrates that the level freezing is independent of the detection or nondetection of the emitted photon in a real photon-detector. Instead it is the possibility of the irreversible decay into vacuum caused by the interaction with the environment, which is responsible for the level freezing.

The fact that we are dealing with null measurements establishes another difference compared to the Itano et al. experiment. There the probability has been investigated of finding the initial state at time T regardless of the past history of the system in the interval $[0,T]$. This is to be described by $P^Z(T)$ given in Eq. (10). But, as has been pointed out in [11], this is not the genuine quantum Zeno effect according to Misra and Sudarshan, which is based on the idea of permanent survival as it is expressed by $P^Z_{ps}(T)$ of Eq. (12). This needs continuous observations as they can be found in our proposal.

4. Treatment within a phenomenological theory of continuous measurements

The level freezing which we have observed above is a realization of a genuinely continuous (strictly not sequential) measurement of energy, which has led to one uninterrupted null-result of duration T. This represents a particular measurement readout $[E]$, namely $E(t)=E_1=\text{const.}$, with $0\le t\le T$. There are different approaches to the theory of continuous quantum measurements (for a review see [12]). We will refer to the presentation in [13] (for the realization scheme see [14]). Note that the assumption of instantaneous wave function collapse is not at all necessary in a phenomenological theory of continuous quantum measurement. Our phenomenological approach for example goes back to restricted path integrals. We call it the method of the complex Hamiltonian (mcH). This fully elaborated scheme allows the treatment of all strengths of the influence of the measurement on the measured system from weak to strong.

According to the mcH, a system with the Hamiltonian H_0+V, subject to the continuous measurement of energy resulting in the readout $[E]=\{E(t)\}$, is described by the Schrödinger equation with the effective Hamiltonian containing an imaginary part:

$$H_{[E]}=H_0+V-i\kappa\big(H_0-E(t)\big)^2, \tag{15}$$

where the constant κ characterizes the strength of the measurement. The inverse of this constant is a measure of the measurement fuzziness.

For comparison with the mcH we introduce a 2-level Hamiltonian H_0 with eigenvalues $E_i=\hbar\omega_i$ and rewrite our Hamiltonian H_γ which describes the system without the driving field, in the form

$$H_\gamma=H_0-i\hbar\gamma|2\rangle\langle 2|. \tag{16}$$

Then the complete Hamiltonian $H_\gamma+V$ used in Section 2 for the description of our system is identical to Eq. (15) provided that $E(t)\equiv E_1$ and the coefficient κ is expressed by the level width γ and the energy difference of the levels $\Delta E=\hbar(\omega_2-\omega_1)$ as follows: $\kappa=\gamma/\Delta E^2$. Notice that the measurement readout $E(t)\equiv E_1$ means in the mcH that the system, according to the measurement, is staying on the level $|1\rangle$ all the time. Thus, there is complete agreement between the results of mcH and our dynamical consideration of Section 2.

The so called level resolution time $T_{lr}=1/(\kappa\Delta E)$ is a measure of the weakness of the continuous measurement and of the resulting fuzziness of the readout. It is in our case $T_{lr}=\gamma^{-1}$. The condition $T_l\ll\Omega^{-1}$ characterizes what is called the Zeno regime (strong influence of the measurement). This agrees with the condition used above in Sections 2, 3. The comparison with the phenomenological approach mcH has enabled us to introduce the concept of the weakness of our continuous null measurement and to fix it quantitatively. H_γ refers to the particular readout $[E]=E_1$. According to the mcH the probability to obtain this $[E]$ agrees with $|a_1(T)|^2$ of Eq. (9).

5. Similar systems

Some systems resembling the one considered above have been discussed in the literature.

In the papers [15,16] continuous null measurements of transitions in a 2-level 'atom' ($|1\rangle,|2\rangle$) were considered. The atom occupies level $|1\rangle$ at the initial moment of time, its evolution without observation is described by the equation $|\dot\psi\rangle=-i\beta\hat\sigma_1|\psi\rangle$, thus the atom oscillates between two levels with frequency β. The atom is exposed to a continuous observation by an apparatus indicating the transition from level $|1\rangle$ to level $|2\rangle$. The apparatus is coupled only to the state $|2\rangle$ of the atom by an interaction Hamiltonian of the form $|2\rangle\langle 2|\otimes H$, where H is a selfadjoint operator in the Hilbert space of the apparatus. This coupling is assumed to switch on a process in the apparatus resulting in the quick indication of the transition. Neglecting the inner dynamical evolution of the apparatus, the authors showed that the evolution of the system is inhibited if the apparatus is sufficiently fast. Their conclusion is that the Zeno effect may take place and that it can be considered as a consequence of pure dynamical unitary evolution of the combined system object-apparatus. The scheme of observation considered in [16] is however rather abstract and no experimental realization of this scheme has been discussed.

The 3-level scheme identical to the one considered by us has been mentioned in the work of Plenio et al. [17] in connection with the Zeno effect but with regard to a quite different aspect. Without any detailed consideration, it was noted that the oscillatory behavior of probability P_1 is inhibited and replaced by the exponential one if level $|2\rangle$ spontaneously decays to another state during a sufficiently short time. This feature of the system was related to a finite width of the level $|2\rangle$, but the inhibition of oscillations was not connected with the quantum Zeno effect. On the contrary, the authors suggested to use the resulting behavior of level $|1\rangle$ as a model for a decaying system which shows a long enough non-exponential period. This 'artificial' decaying system had then to be measured repeatedly to demonstrate the Zeno effect.

Rabi oscillations with radiative damping have been studied in different contexts. We give an example: The state $|1\rangle$ is considered to be the ground state. A higher state $|2\rangle$ is assumed to be unstable. The system decays back to the state $|1\rangle$ with emission of a photon. Investigation of such a system produces the well known result, that Rabi oscillations are damped by radiative decay. If the decay rate of level $|2\rangle$ is much greater than the Rabi frequency the probability to find the system on the level $|1\rangle$ remains close to unity for all times. In this case (in contrast to our 3-level system) there are simultaneously two mechanisms of 'freezing' the system on the level $|1\rangle$: the fast transition from level $|2\rangle$ back to level $|1\rangle$ and in addition Zeno-like inhibition of Rabi oscillations connected with broadening of the level $|2\rangle$. Stimulated and spontaneous emission happen together. It is difficult to separate the two mechanisms. We cannot interpret the resulting behavior as Zeno effect, because the two possible observational results – that a photon is or is not emitted – are both related to the same information, namely that the system is transferred to level $|1\rangle$. This example shows that not any level 'freezing' may be taken as an indication for the presence of a Zeno type measurement influence. In fact the analysis must be carried out the other way round: If a situation can be interpreted as a measurement of Zeno type (including null measurements) than level freezing is to be expected.

6. Conclusions

We have shown that the freezing on the initial level (inhibition of a transition) caused by the possibility of spontaneous decay of the other level can indeed be understood as a Zeno effect because it goes back to the measurement of the system. This is supported by a comparison with the phenomenological scheme based on repeated projection measurements and the phenomenologically described continuous energy measurement in the Zeno regime. After appropriate adjustment of parameters they all agree with the result of the quantum-optical calculation.

The Zeno effect as described above is a particular quantum phenomenon related to quantum measurements which may occur in many different physical situations. To refer to it is of great predictive power: before any involved microphysical calculation has been performed, it must be expected that freezing on a level (or at least slowing down of a transition) will happen, if measurement influences the system.

Referring to our case, we know that quantum measurement may be explained as an interaction with the environment. The spontaneous emission is the mediator between the 2-level system and the environment. It suggests itself to make use of this by treating conversely the influence of the environment as measurement of the system. This is what we have done above. As shown, the results are in accord with the microphysical calculation. The simplicity and the usefulness of the approach became evident.

The advantage of the particular system presented above is, that it is easier to analyse than the standard example of Itano et al. [4] and shows nevertheless to a certain extent a counterintuitive behavior. At the same time it is a very simple example of a null measurement, which is sometimes not quite correctly called interaction free measurement. Potentially the most interesting aspect of the system may be that it represents a truly continuous measurement, which can easily be handled and which we have discussed above only in its Zeno limit.

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