ELSEVIER

Physica A 287 (2000) 193–205

PHYSICA A

www.elsevier.com/locate/physa

Quantum Zeno-like effect and spectra of particles in cascade transition

Alexander D. Panov

Russian Research Center "Kurchatov's Institute", 123182 Kurchatov Sq., Moscow, Russia

Received 11 January 2000

Abstract

Shrödinger equation for two-step spontaneous cascade transition in a three-level quantum system is solved by means of Markovian approximation for non-Markovian integro-differential evolution equations for amplitudes of states. It is shown that both decay constant and radiation shift of initial level are affected by instability of intermediate level of the cascade. These phenomena are interpreted as the different manifestations of quantum Zeno-like effect. The spectra of particles emitted during the cascade transition are calculated in the general case and, in particular, for an unusual situation when the initial state is lower than the intermediate one. It is shown that the spectra of particles do not have a peak-like shape in the latter case.

PACS: 03.65.Bz; 32.70.Jz

Keywords: Quantum measurement theory; Zeno effect; Spontaneous decay

1. Introduction

The term "quantum Zeno paradox" had been introduced in Refs. [1,2]. It was argued there that an unstable particle which was continuously observed in order to see whether it decays would never be found to decay (for review see Ref. [3]). In the present paper we restrict our consideration to the special case of continuous waiting-mode (or negative result) observations of spontaneous decay. An example of such measurements is a registration, by a permanently presented detector, of particles emitted during quantum state decay. Until the detector is "discharged", we continuously obtain information that the system is in the initial excited state. Another interesting example of waiting-mode observation of spontaneous decay one can find in Ref. [4]. It was shown [5] that in

E-mail address: a.panov@relcom.ru (A.D. Panov).

0378-4371/00/$ - see front matter
PII: S0378-4371(00)00316-2

the general case decay may be "frozen" by continuous waiting-mode observation only at the limit of infinitely fast reaction of measuring device on event of decay (infinitely short decoherence time). However, in case of realistic decoherence time the decay may be perturbed in various directions: it may be either slowed down or fastened. The sign of the effect depends on details of transition matrix elements behavior and on transition energy. This is a quantum Zeno effect, not a paradox. Thus, in our definition, the quantum Zeno effect is any influence of continuous measurement to probability of decay. At the limit of infinitely fast measuring device quantum Zeno effect means "freezing" of decay. Qualitatively, the complicated behavior of quantum Zeno effect may be related to a complicated behavior of initial part of decay curve. We will discuss this relation in more details elsewhere.

It is a difficult problem to determine how a permanently presented detector affects on the probability of decay in realistic situations [5]. But the kinematics of this process are very similar to those of a group of phenomena which was called quantum Zeno-like effects [6]. All these phenomena (including above-mentioned observation) are described by the same general equation for perturbation of decay probability [6]. Also, these phenomena has a common main feature: The final state of decay could not be considered as stable, but further transition to other orthogonal states occur. Just these transitions perturb the decay constant. The consideration of some of Zeno-like effects turns out to be much simpler than the genuine Zeno effect. So, it is reasonable to begin with more simple problems.

Systems that show Zeno-like effects differ from each other by the reason of transition from final state of decay. The system with forced resonance transition from the final state of decay was studied in Refs. [7–9]. The analogous system was considered as a particular case of a general Zeno-like system in Ref. [6]. Mensky [10] was the first who proposed to consider a spontaneously decaying system with a spontaneously decaying final state as the system demonstrating quantum Zeno or Zeno-like effect. A simple example of such a system is the system with two-step spontaneous cascade transition. Such systems are the subject of the present paper.

Let X be a three-level system (for example, atom or atomic nucleus) with states $|x_0\rangle$, $|x_1\rangle$, $|x_2\rangle$ and eigenenergies ω_0^x, ω_1^x, ω_2^x, respectively (it is assumed $\hbar = 1$ hereafter). Suppose that system X was prepared in state $|x_0\rangle$ at the initial moment of time $t = 0$. State $|x_0\rangle$ is unstable and decays spontaneously to state $|x_1\rangle$ due to interaction with another system ("field"). The latter system has a continuous spectrum of states. Let state $|x_1\rangle$ be also unstable. System X further decays from state $|x_1\rangle$ to final stable state $|x_2\rangle$. We suppose for simplicity that direct transition from state $|x_0\rangle$ to $|x_2\rangle$ is forbidden. Such a system exhibits cascade spontaneous transition from state $|x_0\rangle$ to state $|x_2\rangle$ through intermediate state $|x_1\rangle$. This phenomenon was studied in some detail many years ago [11–13] and was discussed in classical monographs [14,15]. A new property of cascade transition that was pointed out in Ref. [10] was that the instability of level $|x_1\rangle$ should affect the life-time of level $|x_0\rangle$.

It was noted in Ref. [10] that during cascade transition the second fast transition $|x_1\rangle \to |x_2\rangle$ after decay of initial state $|x_0\rangle$ to $|x_1\rangle$ was similar to waiting-mode

observation of first decay $|x_0\rangle \to |x_1\rangle$. The main difference of the second transition from a genuine continuous measurement is that it is not possible to switch off the interaction causing the second transition, but it is possible to stop a measurement. Hence, the perturbation of decay rate of transition $|x_0\rangle \to |x_1\rangle$ by instability of state $|x_1\rangle$ may be attributed to quantum Zeno-like effect. We use the term quantum Zeno effect as synonym for quantum Zeno-like effect throughout the present paper.

The formula for decay rate of state $|x_0\rangle$ perturbed by instability of state $|x_1\rangle$ was derived in Ref. [6]. With notations analogous to those in our paper, this formula reads as

$$\tilde{\Gamma}_0 = 2\pi \int_0^\infty d\omega \, \mathscr{V}(\omega) \frac{1}{\pi} \frac{\lambda_1}{\lambda_1^2 + (\omega - \omega_{01} + \mu_1)^2} \,. \tag{1}$$

Here $\omega_{01} = \omega_0^x - \omega_1^x$; $\mathscr{V}(\omega)$ is the sum of all square modula of transition matrix elements related to the same energy ω of emitted particle; λ_1 is the real part of decay constant of level $|x_1\rangle$; μ_1 is the contribution to radiation shift of level $|x_1\rangle$ from discrete level $|x_2\rangle$ [16]. We shall hereafter mention similar energy shifts as radiation shifts simply. The complex decay constant γ_1 of level $|x_1\rangle$ is $\gamma_1 = \lambda_1 + i\mu_1$ if the system was prepared in state $|x_1\rangle$. At the limit of $\lambda_1 \to 0$ Eq. (1) transforms into conventional Fermi's Golden rule

$$\Gamma_0 = 2\pi \mathscr{V}(\omega_{01} - \mu_1) \tag{2}$$

but wherein transition energy is corrected by radiation shift of level $|x_1\rangle$. However, deviation of perturbed probability $\tilde{\Gamma}_0$ from unperturbed value Γ_0 exists, if $\lambda_1 \neq 0$. Just this phenomenon is considered as quantum Zeno effect in Ref. [10]. It is easily seen from Eq. (1) that Zeno effect is strong if λ_1 is comparable with ω_{01} by its value. If formally $\lambda_1 \to \infty$ we obtain $\tilde{\Gamma}_0 = 0$. This is pure "quantum Zeno paradox".

It is possible to make an interesting conclusion from Eq. (1). Suppose $\omega_{01} - \mu_1 < 0$. Then Golden rule Eq. (2) predicts zero probability of decay of level $|x_0\rangle$ since $\mathscr{V}(\omega) \equiv 0$ for $\omega < 0$. There are no field quanta with negative energy. However, Eq. (1) shows that perturbed value of probability of decay $\tilde{\Gamma}_0$ is greater than zero in this situation generally. This phenomenon is a special case of quantum Zeno effect. Hence, the transition from lower ($|x_0\rangle$) to upper ($|x_1\rangle$) level is possible, and positive energy quanta should be emitted in such a process. So, an important question arises: What are the spectra of quanta emitted during transitions $|x_0\rangle \to |x_1\rangle$ and $|x_1\rangle \to |x_2\rangle$ in this unusual situation? Also, what are these spectra in the general case when transition energy ω_{01} is comparable with decay constant λ_1 of level $|x_1\rangle$? Obviously, these spectra cannot be Lorentzian-shape peaks.

Eq. (1) was derived on the base of the second-order perturbation theory in Ref. [6]. The spectra of emitted particles cannot be calculated by this method. So, the present paper includes two aims. Firstly, we derive no-decay amplitude of initial state $|x_0\rangle$ and probability $\tilde{\Gamma}_0$ by nonperturbative method which is based on direct transition from non-Markovian evolution equations to Markovian approximation. Secondly, we obtain all spectra of interest from our nonperturbative solution of Shrödinger equation: mutual energy distribution of particles emitted in the first and second transitions,

spectra of particles emitted in the first and second transitions separately, and distribution of the sum of energy of first and second emitted particles. Some of these spectra was determined early [15], but those results are not related to the conditions $\omega_{01} \sim \lambda_1$ or $\omega_{01} < 0$.

This paper is organized as follows. In Section 2 we discuss the Markovian approximation for a spontaneous exponential decay in two-level system. In Section 3 we use the formalism developed in Section 2 for description of cascade transition in a three-level system and obtain our main results: perturbed values of decay constant and radiation shift of level $|x_0\rangle$ and spectra of particles. Finally, some features of these results are discussed in Section 4 and conclusions are drawn.

2. Markovian approximation in two-level problem

We consider a model of spontaneous transition of a general type. Let X be a two-level system $(|x_0\rangle, |x_1\rangle)$. System X interacts with another system F (field). System F has a ground state $|f_0\rangle$ and continuous spectrum of exited states $|y_\eta\rangle$, where η is the index of state in the continuous spectrum. Since we discuss in Section 3 a field with quanta of two different kinds, we use the notation $|y\rangle$ for field quanta instead of $|f\rangle$. The continuous spectrum states are normalized by condition $\langle y_\eta | y_{\eta'}\rangle = \delta(\eta - \eta')$. Let the initial state of the combined system $X \otimes F$ at time $t = 0$ be

$$|\Psi(0)\rangle = |x_0\rangle \otimes |f_0\rangle \equiv |x_0 f_0\rangle \,.$$

The Hamiltonian of system is

$$H = H_0 + V \,,$$

where H_0 is a "free" Hamiltonian

$$H_0 = \omega_0^x |x_0\rangle\langle x_0| + \omega_1^x |x_1\rangle\langle x_1| + \int \omega_\eta^y b_\eta^+ b_\eta \, \mathrm{d}\eta$$

and V is an interaction between X and F:

$$V = \int [v(\eta) b_\eta^+ |x_1\rangle\langle x_0| + v^*(\eta) b_\eta |x_0\rangle\langle x_1|] \, \mathrm{d}\eta \,. \tag{3}$$

In Eq. (3) b_η^+ is the creation operator for state $|y_\eta\rangle$. It does not matter what is the commutation relation for operators b_η: either $[b_\eta, b_{\eta'}^+]_- = \delta(\eta - \eta')$ or $[b_\eta, b_{\eta'}^+]_+ = \delta(\eta - \eta')$. It is easy to see that $v(\eta)$ is a matrix element of transition: $v(\eta) = \langle x_1 y_\eta | V | x_0 f_0\rangle$.

To solve the Shrödinger equation

$$|\dot{\Psi}(t)\rangle = -\mathrm{i}(H_0 + V)|\Psi(t)\rangle \tag{4}$$

we use the ansatz

$$|\Psi(t)\rangle = |x_0 f_0\rangle a_0(t) \mathrm{e}^{-\mathrm{i}\omega_0^x t} + \int \mathrm{d}\eta \, |x_1 y_\eta\rangle a_{1\eta}(t) \mathrm{e}^{-\mathrm{i}(\omega_1^x + \omega_\eta^y)t} \,. \tag{5}$$

Substituting Eq. (5) for $|\Psi(t)\rangle$ in Eq. (4) we obtain the system of equations:

$$\dot{a}_0(t) = -\mathrm{i}\int \mathrm{d}\eta\, v^*(\eta)\mathrm{e}^{-\mathrm{i}(\omega_\eta^y-\omega_{01})t}a_{1\eta}(t)\,, \tag{6}$$

$$\dot{a}_{1\eta}(t) = -\mathrm{i}v(\eta)\mathrm{e}^{\mathrm{i}(\omega_\eta^y-\omega_{01})t}a_0(t)\,. \tag{7}$$

Eq. (7) can be solved for coefficients $a_{1\eta}$. Substituting the solution for $a_{1\eta}$ in Eq. (6) we get the equation for coefficient $a_0(t)$:

$$\dot{a}_0(t) = -\int_0^t a_0(t_1)q_0(t-t_1)\,\mathrm{d}t_1\,, \tag{8}$$

where

$$q_0(\tau) = \int |v(\eta)|^2\mathrm{e}^{-\mathrm{i}(\omega_\eta^y-\omega_{01})\tau}\,\mathrm{d}\eta\,. \tag{9}$$

The amplitude $a_0(t)$ is a solution of non-Markovian equation (8): the derivative of $\dot{a}_0$ at the moment of time t is expressed through all values of a_0 for all moments of time from 0 to t.

Sudbery [17,18] proposed qualitative arguments that the function $q_0(\tau)$ in Eq. (8) was a very narrow peak around value $\tau = 0$ for usual decay systems. Besides, it is possible to understand why it should be the case if we consider the behavior of function $v(\eta)$.

The index η of state $|y_\eta\rangle$ can be represented as the eigenenergy of the state ω^y and the degeneration index α^y: $\eta = \{\omega^y, \alpha^y\}$. Then Eq. (9) can be rewritten as

$$q_0(\tau) = \mathrm{e}^{\mathrm{i}\omega_{01}\tau}\int_0^\infty \mathscr{V}(\omega^y)\mathrm{e}^{-\mathrm{i}\omega^y\tau}\,\mathrm{d}\omega^y\,, \tag{10}$$

where

$$\mathscr{V}(\omega^y) = \int |v(\omega^y,\alpha^y)|^2\,\mathrm{d}\alpha^y\,. \tag{11}$$

The integral in Eq. (11) means a sum for discrete indexes. Eq. (10) shows that the function $q_0(\tau)$ is a Fourier transform of function $\mathscr{V}(\omega^y)$ up to factor $\exp(\mathrm{i}\omega_{01}\tau)$ which is equal to one by module. The function $\mathscr{V}(\omega^y)$ is very wide for usual decay systems. For example, in the case of electromagnetic 2P–1S transition of hydrogen atom, the value Λ of natural cut-off of function $\mathscr{V}(\omega^y)$ is $\Lambda = \frac{3}{2}\alpha m_e \approx 5.6\times 10^3\,\mathrm{eV}$, where α is the fine structure constant and m_e is the electron mass [19–21]. The value of Λ is much greater than the energy of 2P–1S transition. The Fourier transform of wide real non-negative function $\mathscr{V}(\omega^y)$ is a narrow peak near $\tau = 0$. Consequently, the function $q_0(\tau)$ is a narrow peak near $\tau = 0$ too. The width of this peak is about $\tau_{\mathrm{Zen}} = 1/\Lambda$. Suppose $a_0(t)$ to vary slowly during time intervals of the order of $1/\omega_{01}$ for times $t \gg 1/\omega_{01}$. Then, for the same times, $a_0(t)$ is approximately constant during the time interval of order τ_{Zen} and the function $a_0(t-t_1)$ may be moved out from the integral in Eq. (8) at time t. Making also the variable change $\tau = t - t_1$ we rewrite Eq. (8) as

$$\dot{a}_0(t) = -a_0(t)\int_0^t q_0(\tau)\,\mathrm{d}\tau\,. \tag{12}$$

Therefore, we obtain an approximate Markovian equation (12) for $a_0(t)$ instead of non-Markovian equation (8). Recall that Eq. (12) is valid only for $t \gg 1/\omega_{01}$. It is not difficult to calculate the integral in r.h.s. of Eq. (12) using Eq. (10):

$$\int_0^t q_0(\tau)\,\mathrm{d}\tau = \int_0^\infty \mathscr{V}(\omega^y)\left[\frac{\sin(\omega^y-\omega_{01})t}{\omega^y-\omega_{01}} - \mathrm{i}\frac{1-\cos(\omega^y-\omega_{01})t}{\omega^y-\omega_{01}}\right]\mathrm{d}\omega^y\,. \tag{13}$$

Suppose $\mathscr{V}(\omega^y)$ is sufficiently smooth. Then it is seen that the integral in the r.h.s. of Eq. (13) does not depend on time for $t \gg 1/\omega_{01}$. Hence, we can change the upper limit of integral from t to infinity and find that

$$\int_0^\infty q_0(\tau)\,\mathrm{d}\tau = \gamma_0 = \lambda_0 + \mathrm{i}\mu_0\,,$$

where

$$\lambda_0 = \mathrm{Re}\,\gamma_0 = \pi\mathscr{V}(\omega_{01})\,,$$
$$\mu_0 = \mathrm{Im}\,\gamma_0 = -P\int_0^\infty \frac{\mathscr{V}(\omega^y)}{\omega^y-\omega_{01}}\,\mathrm{d}\omega^y\,. \tag{14}$$

Here P denotes the principal value of an integral.

Eq. (12) reads now as $\dot{a}_0(t) = -\gamma_0 a_0(t)$ and has the solution $a_0(t) = \exp(-\gamma_0 t)$. This is the usual exponential decay law. The real part of γ_0 determines the probability of decay per unit of time $\Gamma_0 = 2\,\mathrm{Re}\,\gamma_0 = 2\pi\mathscr{V}(\omega_{01})$; the imaginary part of γ_0 is the radiation shift of level $|x_0\rangle$.

3. Decay constants, radiation shifts, and spectra in two-step cascade transition

We consider three-level system X with cascade transition $|x_0\rangle \to |x_1\rangle \to |x_2\rangle$ in this section. The transitions result from interaction of system X with another system F (field). Suppose that two different types of quanta are emitted during the first and second transition. The quanta $|y_\eta\rangle$ are created during transition $|x_0\rangle \to |x_1\rangle$ and the quanta $|z_\zeta\rangle$ are created during transition $|x_1\rangle \to |x_2\rangle$. The y and z quanta have creation operators b_η^+ and c_ζ^+, respectively. We admit that for all η and ζ the operators b_η and c_ζ satisfy the relation

$$[b_\eta, c_\zeta^+]_- = 0, \quad [b_\eta, c_\zeta]_- = 0\,. \tag{15}$$

Eq. (15) represents the meaning of difference between particles y and z. Operators b_η may be either Bozonic or Fermionic type, the same is true for operators c_ζ. The statistics type of y and z particles may be different from each other. For example, we may consider cascade nuclear transition when an atomic electron is emitted in the first transition (the inner nuclear conversion phenomenon) and electromagnetic quantum is emitted in the second one. Our model is correct for this case. We also can consider a cascade electromagnetic transition, but the energy of the first transition is much less than the energy of the second one. In this case Eq. (15) is not strictly true for all quanta

of such cascade transition, but our model may be accounted as a good approximation in this case as well. The generalization to the case when we cannot distinguish between quanta emitted in the first transition and in the second transition is not straightforward and is not discussed here.

The Hamiltonian of the system $X \otimes F$ is

$$H = H_0 + V + W ,$$

where

$$H_0 = \sum_{\xi=0}^{2} \omega_\xi^x |x_\xi\rangle\langle x_\xi| + \int \mathrm{d}\eta\, \omega_\eta^y b_\eta^+ b_\eta + \int \mathrm{d}\zeta\, \omega_\zeta^z c_\zeta^+ c_\zeta , \tag{16}$$

$$V = \int \mathrm{d}\eta [v(\eta) b_\eta^+ |x_1\rangle\langle x_0| + v^*(\eta) b_\eta |x_0\rangle\langle x_1|] , \tag{17}$$

$$W = \int \mathrm{d}\zeta [w(\zeta) c_\zeta^+ |x_2\rangle\langle x_1| + w^*(\zeta) c_\zeta |x_1\rangle\langle x_2|] . \tag{18}$$

The notations in Eqs. (16)–(18) are similar to those of Section 2 and obvious. For initial state $|\Psi(0)\rangle = |x_0 f_0\rangle$ we solve the Shrödinger equation

$$|\dot{\Psi}(t)\rangle = -\mathrm{i}(H_0 + V + W)|\Psi(t)\rangle \tag{19}$$

using the ansatz

$$|\Psi(t)\rangle = |x_0 f_0\rangle a_0(t) \mathrm{e}^{-\mathrm{i}\omega_0^x t} + \int \mathrm{d}\eta\, |x_1 y_\eta\rangle a_{1\eta}(t) \mathrm{e}^{-\mathrm{i}(\omega_1^x + \omega_\eta^y)t}$$

$$+ \int \mathrm{d}\eta \int \mathrm{d}\zeta |x_2 y_\eta z_\zeta\rangle a_{2\eta\zeta}(t) \mathrm{e}^{-\mathrm{i}(\omega_2^x + \omega_\eta^y + \omega_\zeta^z)t} . \tag{20}$$

Substituting Eq. (20) for $|\Psi(t)\rangle$ in Eq. (19) we obtain the system of equations

$$\dot{a}_0(t) = -\mathrm{i} \int \mathrm{d}\eta\, v^*(\eta) \mathrm{e}^{-\mathrm{i}(\omega_\eta^y - \omega_{01})t} a_{1\eta}(t) , \tag{21}$$

$$\dot{a}_{1\eta}(t) = -\mathrm{i} v(\eta) \mathrm{e}^{\mathrm{i}(\omega_\eta^y - \omega_{01})t} a_0(t) - \mathrm{i} \int \mathrm{d}\zeta\, w^*(\zeta) \mathrm{e}^{-\mathrm{i}(\omega_\zeta^z - \omega_{12})t} a_{2\eta\zeta}(t) , \tag{22}$$

$$\dot{a}_{2\eta\zeta}(t) = -\mathrm{i} w(\zeta) \mathrm{e}^{\mathrm{i}(\omega_\zeta^z - \omega_{12})t} a_{1\eta}(t) . \tag{23}$$

Here $\omega_{ij} = \omega_i^x - \omega_j^x$. From Eq. (23) we have

$$a_{2\eta\zeta}(t) = -\mathrm{i} w(\zeta) \int_0^t \mathrm{d}t_1\, \mathrm{e}^{\mathrm{i}(\omega_\zeta^z - \omega_{12})t_1} a_{1\eta}(t_1) . \tag{24}$$

Substituting Eq. (24) for $a_{2\eta\zeta}(t)$ in Eq. (22), we obtain a non-Markovian equation for coefficients $a_{1\eta}$:

$$\dot{a}_{1\eta}(t) = -\mathrm{i} v(\eta) \mathrm{e}^{\mathrm{i}(\omega_\eta^y - \omega_{01})t} a_0(t) - \int_0^t \mathrm{d}t_1\, a_{1\eta}(t_1) q_1(t - t_1) , \tag{25}$$

where

$$q_1(\tau) = e^{i\omega_{12}\tau} \int \mathcal{W}(\omega^z) e^{-i\omega^z\tau}\, d\omega^z\,, \tag{26}$$

$$\mathcal{W}(\omega^z) = \int |w(\omega^z, \alpha^z)|^2\, d\alpha^z \tag{27}$$

and we supposed $\zeta = \{\omega^z, \alpha^z\}$. The equations (26) and (27) are quite similar to equations (10) and (11) (Section 2). The only difference is that Eqs. (10) and (11) are related to transition $|x_0\rangle \to |x_1\rangle$ but Eqs. (26) and (27) are related to transition $|x_1\rangle \to |x_2\rangle$. The integral in r.h.s. of Eq. (25) is similar to the integral in r.h.s. of Eq. (8). Therefore, arguing as in Section 2, we see that Eq. (25) may be changed by approximate Markovian equation

$$\dot{a}_{1\eta}(t) = -iv(\eta) e^{i(\omega_\eta^y - \omega_{01})t} a_0(t) - \gamma_1 a_{1\eta}(t)\,, \tag{28}$$

where

$$\gamma_1 = \lambda_1 + i\mu_1 = \int_0^\infty q_1(\tau)\, d\tau\,,$$

$$\lambda_1 = \pi \mathcal{W}(\omega_{12}), \qquad \mu_1 = -P \int_0^\infty \frac{\mathcal{W}(\omega^z)}{\omega^z - \omega_{12}}\, d\omega^z\,.$$

The solution of Eq. (28) is

$$a_{1\eta}(t) = -iv(\eta) \int_0^t e^{-\gamma_1(t-t_1)} e^{i(\omega_\eta^y - \omega_{01})t_1} a_0(t_1)\,. \tag{29}$$

Substituting Eq. (29) for $a_{1\eta}(t)$ in Eq. (21) we find the equation for amplitude $a_0(t)$:

$$\dot{a}_0(t) = -\int_0^t a_0(t_1)\tilde{q}_0(t - t_1)\, dt_1\,, \tag{30}$$

where

$$\tilde{q}_0(\tau) = e^{-\gamma_1\tau} e^{i\omega_{01}\tau} \int \mathcal{V}(\omega^y) e^{-i\omega^y\tau}\, d\omega^y\,. \tag{31}$$

The tilde indication of function $\tilde{q}_0(\tau)$ means that this function is related to transition $|x_0\rangle \to |x_1\rangle$ perturbed by instability of state $|x_1\rangle$. Further the meaning of tilde will be the same in all cases. Function $\tilde{q}_0(\tau)$ differs from nondisturbed function $q_0(\tau)$ Eq. (10) by additional factor $\exp(-\gamma_1\tau)$. The module of this factor is a decreasing function since $\mathrm{Re}\,\gamma_1 = \lambda_1 > 0$. Consequently, the function $\tilde{q}_0(\tau)$ is a narrow peak near the value $\tau = 0$ as well as the nondisturbed function $q_0(\tau)$ (see Section 2). Hence, we can change the non-Markovian equation (30) to Markovian one

$$\dot{a}_0(t) = -\tilde{\gamma}_0 a_0(t)\,, \tag{32}$$

where

$$\tilde{\gamma}_0 = \tilde{\lambda}_0 + i\tilde{\mu}_0 = \int_0^\infty \tilde{q}_0(\tau)\, d\tau\,. \tag{33}$$

It is not difficult to obtain from Eq. (33) and Eq. (31):

$$\tilde{\lambda}_0 = \pi \int_0^\infty \mathscr{V}(\omega^y) \frac{1}{\pi} \frac{\lambda_1}{\lambda_1^2 + (\omega^y - \omega_{01} + \mu_1)^2} \, \mathrm{d}\omega^y \,, \tag{34}$$

$$\tilde{\mu}_0 = \int_0^\infty \mathscr{V}(\omega^y) \frac{\omega^y - \omega_{01} + \mu_1}{\lambda_1^2 + (\omega^y - \omega_{01} + \mu_1)^2} \, \mathrm{d}\omega^y \,. \tag{35}$$

Solving Eq. (32), we get

$$a_0(t) = \mathrm{e}^{-\tilde{\gamma}_0 t} \,. \tag{36}$$

It follows from Eq. (36) that $\tilde{\gamma}_0$ is the complex decay constant of state $|x_0\rangle$. Decay constant is perturbed by instability of state $|x_1\rangle$. Thus, the probability of decay per unit of time is $\tilde{\Gamma}_0 = 2\mathrm{Re}\,\tilde{\gamma}_0 = 2\tilde{\lambda}_0$. This value coincides with the result obtained early in Ref. [6] by perturbation method (comp. Eqs. (1) and (34)).

Now let us find the spectra of particles y and z created during the first and second transitions of system X. These spectra are defined by values $|a_{1\eta}(t)|^2$ and $|a_{2\eta\zeta}(t)|^2$ as $t \to \infty$. Substituting Eq. (36) for $a_0(t)$ in Eq. (29), we obtain

$$a_{1\eta}(t) = -\mathrm{i}v(\eta) \frac{\mathrm{e}^{[\mathrm{i}(\omega_\eta^y - \omega_{01}) - \tilde{\gamma}_0]t} - \mathrm{e}^{-\gamma_1 t}}{\mathrm{i}(\omega_\eta^y - \omega_{01}) + \gamma_1 - \tilde{\gamma}_0} \,. \tag{37}$$

It is readily seen that

$$\lim_{t\to\infty} |a_{1\eta}(t)|^2 = 0 \,.$$

This means that coefficients $a_{1\eta}(t)$ do not contribute to spectra of particles. This could be expected because these coefficients relate to intermediate state of the system.

Substituting Eq. (37) for $a_{1\eta}(t)$ in Eq. (24), we get the expression for $a_{2\eta\zeta}(t)$:

$$a_{2\eta\zeta}(t) = -\frac{v(\eta)w(\zeta)}{\mathrm{i}(\omega_\eta^y - \omega_{01}) + \gamma_1 - \tilde{\gamma}_0} \times \left\{ \frac{\mathrm{e}^{[\mathrm{i}(\omega_\eta^y + \omega_\zeta^z - \omega_{02}) - \tilde{\gamma}_0]t} - 1}{\mathrm{i}(\omega_\eta^y + \omega_\zeta^z - \omega_{02}) - \tilde{\gamma}_0} - \frac{\mathrm{e}^{[\mathrm{i}(\omega_\zeta^z - \omega_{12}) - \gamma_1]t} - 1}{\mathrm{i}(\omega_\zeta^z - \omega_{12}) - \gamma_1} \right\} \,. \tag{38}$$

It is easy to obtain from Eq. (38) that the limit of $a_{2\eta\zeta}(t)$ as $t \to \infty$ is

$$a_{2\eta\zeta}(\infty) = \frac{v(\eta)w(\zeta)}{[\mathrm{i}(\omega_\eta^y + \omega_\zeta^z - \omega_{02}) - \tilde{\gamma}_0][\mathrm{i}(\omega_\zeta^z - \omega_{12}) - \gamma_1]} \,. \tag{39}$$

Now we can calculate mutual distribution of energy of particles y and z:

$$p(\omega^y, \omega^z) = \int \mathrm{d}\alpha^y \int \mathrm{d}\alpha^z \, |a_{2;\omega^y\alpha^y;\omega^z\alpha^z}(\infty)|^2 \,. \tag{40}$$

From Eq. (39) and Eq. (40) we get

$$p(\omega^y,\omega^z)=\frac{\mathscr{V}(\omega^y)\mathscr{W}(\omega^z)}{[\tilde{\lambda}_0^2+(\omega^y+\omega^z-\bar{\omega}_{02})^2][\lambda_1^2+(\omega^z-\bar{\omega}_{12})^2]}, \tag{41}$$

where $\bar{\omega}_{02}$ and $\bar{\omega}_{12}$ are the corrected values of transition energies

$$\bar{\omega}_{02}=(\omega_0^x+\tilde{\mu}_0)-\omega_2^x, \quad \bar{\omega}_{12}=(\omega_1^x+\mu_1)-\omega_2^x.$$

Let us note that the energy ω_0^x is corrected by perturbed value of radiation shift $\tilde{\mu}_0$ defined by Eq. (35) instead of unperturbed radiation shift Eq. (14).

The spectrum of particles y created in the first transition is defined by

$$p_y(\omega^y)=\int_0^{+\infty} p(\omega^y,\omega^z)\,\mathrm{d}\omega^z=\int_{-\infty}^{+\infty} p(\omega^y,\omega^z)\,\mathrm{d}\omega^z. \tag{42}$$

We change the lower limit of integral in Eq. (42) from 0 to $-\infty$ since $\mathscr{W}(\omega^z)=0$ for all $\omega^z<0$. It is possible to calculate the integral Eq. (42) analytically only if the function $\mathscr{W}(\omega^z)$ is known. In the general case we have to introduce some approximation. Suppose that $|\omega_{01}|\ll\omega_{12}$, $\omega^y\ll\omega_{12}$, and $\mathscr{W}(\omega^z)$ is a sufficiently smooth function:

$$\mathscr{W}(\omega^y+\bar{\omega}_{02})\approx\mathscr{W}(\bar{\omega}_{12})\approx\mathscr{W}(\omega_{12}). \tag{43}$$

Then we can rewrite Eq. (42) as

$$p_y(\omega^y)=\mathscr{V}(\omega^y)\mathscr{W}(\omega_{12})\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega^z}{[\tilde{\lambda}_0^2+(\omega^y+\omega^z-\bar{\omega}_{02})^2][\lambda_1^2+(\omega^z-\bar{\omega}_{12})^2]}. \tag{44}$$

It is not hard to calculate the integral in Eq. (44) by residue theory. We obtain

$$p_y(\omega^y)=\frac{\pi\mathscr{V}(\omega^y)}{\tilde{\lambda}_0}\left[\frac{1}{\pi}\frac{\tilde{\lambda}_0+\lambda_1}{(\tilde{\lambda}_0+\lambda_1)^2+(\omega^y-\bar{\omega}_{01})^2}\right], \tag{45}$$

where $\bar{\omega}_{01}=(\omega_0^x+\tilde{\mu}_0)-(\omega_1^x+\mu_1)$ is the corrected energy of transition $|x_0\rangle\to|x_1\rangle$.

The spectrum of particles z is

$$\begin{aligned}p_z(\omega^z)&=\int_0^{+\infty}p(\omega^y,\omega^z)\,\mathrm{d}\omega^y\\&=\frac{\mathscr{W}(\omega^z)}{\lambda_1^2+(\omega^z-\bar{\omega}_{12})^2}\int_{-\infty}^{+\infty}\frac{\mathscr{V}(\omega^y)\,\mathrm{d}\omega^y}{\tilde{\lambda}_0^2+(\omega^y+\omega^z-\bar{\omega}_{02})^2}.\end{aligned} \tag{46}$$

Suppose $\mathscr{V}(\omega^y)$ to vary slowly during intervals of order $\tilde{\lambda}_0$ for all ω^y. Then we can move $\mathscr{V}(\omega^y)$ out of integral in Eq. (46) for $\omega^y=\bar{\omega}_{02}-\omega^z$. Taking into account also Eq. (43) we have

$$p_z(\omega^z)=\frac{\pi\mathscr{V}(\bar{\omega}_{02}-\omega^z)}{\tilde{\lambda}_0}\left[\frac{1}{\pi}\frac{\lambda_1}{\lambda_1^2+(\omega^z-\bar{\omega}_{12})^2}\right]. \tag{47}$$

Let us find the distribution of the sum of energies $\omega^y+\omega^z=\Omega$. It can easily be checked that

$$p_{y+z}(\Omega)=\int_0^{\Omega}p(\omega^y,\Omega-\omega^y)\,\mathrm{d}\omega^y. \tag{48}$$

It follows from Eq. (48) and Eq. (41) with assumption Eq. (43) that

$$p_{y+z}(\Omega) = \frac{S(\Omega)}{\tilde{\lambda}_0^2 + (\Omega - \bar{\omega}_{02})^2}, \tag{49}$$

where

$$S(\Omega) = \int_0^{\Omega} \mathscr{V}(\omega^y) \left[\frac{1}{\pi} \frac{\lambda_1}{\lambda_1^2 + (\Omega - \omega^y - \bar{\omega}_{12})^2} \right] \mathrm{d}\omega^y.$$

If $\lambda_1 \gg \tilde{\lambda}_0$, function $S(\Omega)$ varies slowly in comparison with the pole-like denominator of Eq. (49). Hence, the spectrum of the sum of particle y and z energies is approximately a narrow Lorentzian-shape peak of width $\tilde{\lambda}_0$ (as could be expected).

4. Discussion and conclusions

The main results of the present paper are as follows:

- Eq. (34) describes perturbed value of the real part of decay constant of level $|x_0\rangle$ (the initial level of cascade transition). The real part is also the half of decay probability per unit of time of level $|x_0\rangle$.
- Eq. (35) describes perturbed value of imaginary part of decay constant of level $|x_0\rangle$. The imaginary part is the perturbed value of radiation shift of level $|x_0\rangle$.
- Eq. (41) describes mutual energy spectrum of particles of the first and of the second transition of a cascade.
- Eq. (45) describes energy spectrum of the first transition of a cascade.
- Eq. (47) describes energy spectrum of the second transition of a cascade.
- Eq. (49) describes distribution of the sum of particle energies created during the first and the second transitions of a cascade.

We discussed Eq. (1) for perturbed value of decay probability $\tilde{\Gamma}_0$ in the Introduction. Since $\tilde{\Gamma}_0 = 2\tilde{\lambda}_0$, so this discussion is related to Eq. (34) as well.

Eq. (35) shows that instability of level $|x_1\rangle$ affects the discrete level contribution to radiation shift of level $|x_0\rangle$ as well as the probability of decay. Therefore, the well-known formula for radiation shift (14) should be replaced by Eq. (35) if λ_1 is comparable with $|\omega_{01}|$. It is easy to see that Eq. (35) transforms into usual Eq. (14) as $\lambda_1 \to 0$, $\mu_1 \to 0$. If formally $\lambda_1 \to \infty$, from Eq. (35) we obtain $\tilde{\mu}_0 \to 0$. This result is similar to $\tilde{\lambda}_0 \to 0$ as $\lambda_1 \to \infty$, therefore, it may be called "an energy-shift quantum Zeno paradox". Similarly, the perturbation of radiation shift of level $|x_0\rangle$ by instability of level $|x_1\rangle$ for realistic values λ_1 may be called "an energy-shift quantum Zeno effect". Note that it could be expected that "energy-shift quantum Zeno effect" would be presented in waiting-mode observation of decay in the general case, not only in cascade transitions. Thus, the same mechanism that perturbs the probability of decay also perturbs the radiation shift of level.

Let us now discuss the expression for particle spectra emitted during the first transition (Eq. (45)) and during the second transition (Eq. (47)). It is suitable to discuss three different situations:

1. If $\lambda_1 \ll \bar{\omega}_{01}$ and $\bar{\omega}_{01} > 0$ then it could be considered that function $\mathscr{V}(\omega^y)$ to vary very slowly in comparison with the pole-like denominators in Eq. (45) and Eq. (47). Hence, we obtain that the spectra defined by Eqs. (45) and (47) are usual Lorentzian-shape peaks. The width of the spectra of first transition is $\tilde{\lambda}_0 + \lambda_1$, but not $\tilde{\lambda}_0$. These conclusions are quite similar to well-known results [15], but λ_0 in Ref. [15] is now changed by perturbed value $\tilde{\lambda}_0$.

2. If $\lambda_1 \sim \bar{\omega}_{01}$ and $\bar{\omega}_{01} > 0$, it cannot be considered that function $\mathscr{V}(\omega^y)$ to vary slowly in comparison with the denominators in Eqs. (45) and (47). Therefore, both the spectra of particles y and z become strongly deformed Lorentzian peaks.

3. Finally, suppose $\bar{\omega}_{01} < 0$. Then the maxima of Lorentzian factors of Eqs. (45) and (47) are positioned in the branch of ω values where $\mathscr{V}(\omega) = 0$. The spectra shapes are defined by the shape of function $\mathscr{V}(\omega)$ multiplied by the tale of Lorentzian peaks now. Therefore, both spectra $p_y(\omega^y)$ and $p_z(\omega^z)$ are continuous rather than peak-like. The energy of quanta y emitted during the first step of cascade transition is positive, of course, in spite of $\bar{\omega}_{01} < 0$.

Thus, the separate spectra of particles y and z may be very wide or even continuous. But it follows from Eq. (49) that the energies of particles y and z remain strongly correlated such that the width of the distribution of sum $\Omega = \omega^y + \omega^z$ is equal to $\tilde{\lambda}_0$ in all cases. This is a manifestation of fundamental uncertainty principle for energy and time.

Acknowledgements

The author acknowledges the fruitful discussions with M.B. Mensky and V.A. Namiot and is grateful to V.A. Aref'ev for the help in preparation of the paper. The work was supported in part by the Russian Foundation of Basic Research, grant 98-01-00161.

References

[1] B. Misra, E.C.G. Sudarshan, J. Math. Phys. 18 (1977) 756.
[2] C.B. Chiu, E.C.G. Sudarshan, B. Misra, Phys. Rev. D 16 (1977) 520.
[3] D. Home, M.A.B. Whitaker, Ann. Phys. 258 (1997) 237.
[4] B. Elattari, S.A. Gurvitz, Effect of the measurement on the decay rate of a quantum system, quant-ph/9908054, 1999.
[5] A.D. Panov, Ann. Phys. 249 (1996) 1.
[6] A.D. Panov, Phys. Lett. A 260 (1999) 441.
[7] E. Mihokova, S. Pascazio, L.S. Schulman, Phys. Rev. A 56 (1997) 25.
[8] S. Pascazio, P. Facchi, Acta Phys. Slovaca 49 (1999) 557.
[9] P. Facchi, S. Pascazio, Spontaneous emission and lifetime modification due to an intense electromagnetic field, quant-ph/9909043, 1999.
[10] M.B. Mensky, Phys. Lett. A 257 (1999) 227.
[11] V. Weisskopf, E. Wigner, Zs. Phys. 63 (1930) 54.

[12] V. Rosenfeld, Zs. Phys. 71 (1931) 273.
[13] H. Casimir, Zs. Phys. 81 (1933) 496.
[14] W. Heitler, The Quantum Theory of Radiation, Clarendon Press, Oxford, 1936, p. 114.
[15] V.B. Berestetskii, E.M. Lifshitz, L.P. Pitaevskii, Landau and Lifshitz Course of Theoretical Physics, Vol. 4, Quantum electrodynamics, Pergamon Press, Oxford, 1982, p. 240.
[16] J. Seke, Physica A 203 (1994) 284.
[17] A. Sudbery, Ann. Phys. 157 (1984) 512.
[18] A. Sudbery, Quantum Mechanics and the Particles of Nature. An Outline for Mathematicians, Cambridge University Press, Cambridge, 1986.
[19] H.E. Moses, Phys. Rev. A 8 (1972) 1710.
[20] J. Seke, Physica A 203 (1994) 269.
[21] P. Facchi, S. Pascazio, Phys. Lett. A 241 (1998) 139.