

# **Effects of the general relativistic spin precessions on the habitability of rogue planets orbiting supermassive black holes**

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## Abstract

Recently, the possibility that several starless telluric planets may form around supermassive black holes (SMBHs) receiving an energy input from the hole’s accretion disk, which, under certain not implausible circumstances, may make them habitable in a terrestrial sense, has gained increasing attention. In particular, an observer on a planet orbiting at distance  $r = 100$  Schwarzschild radii from a maximally rotating Kerr SMBH with mass  $M_{\bullet} = 1 \times 10^8 M_{\odot}$  in a plane slightly outside the equator of the latter would see the gravitationally lensed accretion disk having the same size of the Sun as seen from the Earth. Moreover, the accretion rate might be imagined to be set in such a way that the apparent disk’s temperature would be identical to that of the solar surface. We demonstrate that the post-Newtonian (pN) de Sitter and Lense-Thirring precessions of the spin axis of such a world would rapidly change, among other things, its tilt  $\varepsilon$  to its orbital plane by tens to hundreds of degrees over a time span of, say, just  $\Delta t = 400$  yr, strongly depending on the obliquity  $\eta_{\bullet}$  of the SMBH’s spin to the orbital plane. Thus, such relativistic features would have per se a relevant impact on the long term habitability of the considered planet. It is plausible arguing that much stronger effects would likely alter the sky as seen from the fictional planets appearing in the movie *Interstellar*, which orbit much closer to the event horizon of the SMBH Gargantua.

keywords gravitation – celestial mechanics – (galaxies:) quasars: supermassive black holes – planetary systems – extraterrestrial intelligence

## 1. Introduction

Supermassive black holes (SMBHs) (Melia 2009) are most likely lurking in the center of nearly all large galaxies. The largest ones for which empirical evidences, collected with different techniques, exist are HOLM 15A, whose mass is  $M_{\bullet} = 4.0 \times 10^{10} M_{\odot}$  (Mehrgan et al. 2019), and TON 618 ( $M_{\bullet} = 6.6 \times 10^{10} M_{\odot}$ ) (Shemmer et al. 2004). IC 1101, whose mass has not been measured directly, may even reach the  $M_{\bullet} \leq 10^{11} M_{\odot}$  level (?). A SMBH with  $M_{\bullet} = 6.5 \times 10^9 M_{\odot}$  (Event Horizon Telescope Collaboration et al. 2019a), whose shadow was recently imaged by the Event Horizon Telescope collaboration (Event Horizon Telescope Collaboration et al. 2019b), is located at the center of the supergiant elliptical galaxy M87. Milky Way is believed to host a SMBH with  $M_{\bullet} = 4.15 \times 10^6 M_{\odot}$  (The GRAVITY Collaboration et al. 2019) in Sgr A\* at the Galactic Center. The fictional SMBH known as Gargantua, appeared in the science-fiction movie *Interstellar*, has arguably a mass  $M_{\bullet} = 1 \times 10^8 M_{\odot}$  (Thorne 2014).

It is currently widely accepted that SMBHs are at the basis of the extremely powerful electromagnetic emission of the Active Galactic Nuclei (AGNs) (Fabian 1999) due to the release of gravitational energy of the infalling matter (Madejski 2002).

It was recently pointed out that the accretion disks around SMBHs in low luminosity AGNs (Ricci et al. 2017) may sustain the formation of several telluric, Earth-like rogue (i.e. starless) planets at some parsecs (pc) from them (Wada, Tsukamoto & Kokubo 2019) which, under certain circumstances, may even sustain habitable environments (Lingam, Ginsburg & Bialy 2019). In particular, Wada, Tsukamoto & Kokubo (2019) considered a dusty disk ranging from  $\approx 0.1$  pc to  $\approx 100$  pc around SMBHs with masses  $M_{\bullet} \approx 10^6 - 10^9 M_{\odot}$ . Lingam, Ginsburg & Bialy (2019) considered SMBHs in the mass range  $M_{\bullet} \approx 10^9 - 10^{10} M_{\odot}$  finding that planets at sub-pc distances from the hosting AGNs may become uninhabitable because of complex interactions with the dusty torus as well as strong outflows and winds from the accretion disk. On the other hand, Lingam, Ginsburg & Bialy (2019) warned that the  $\approx 1$  pc threshold should not be regarded as a rigid cutoff because of the remarkable heterogeneity among active galaxies. Moreover, our understanding of their central regions is currently far from being definitive. The zones favourable for prebiotic chemistry and photosynthesis extend up to  $\approx 44$  pc and  $\approx 340$  pc, respectively (Lingam, Ginsburg & Bialy 2019). If  $M_{\bullet} \approx 10^4 - 10^5 M_{\odot}$ , such boundaries reduce below the pc scale (Lingam, Ginsburg & Bialy 2019). Previous, more pessimistic evaluations of the impact of the AGN phase of Sgr A\* in our Galaxy can be found in Balbi & Tombesi (2017). Schnittman (2019) explored some potentially harmful consequences for life on a rogue planet orbiting a SMBH just outside its event horizon as the fictional Miller’s planet in *Interstellar*. Interestingly, Schnittman (2019) noted also that for a planet orbiting a Gargantua-like, maximally spinning SMBH at some 100 Schwarzschild radii<sup>1</sup>  $R_s = 2GM_{\bullet}/c^2$  in a tilted plane, even if only slightly above or below the SMBH’s equatorial plane, the gravitational bending of the electromagnetic waves coming from the assumed Novikov-Thorne type accretion disk (Novikov & Thorne 1973) would make the latter visible as a lensed ring having roughly the same size of the Sun as viewed from the Earth. Furthermore, for a given mass accretion rate, which, from Figure 7 of Schnittman (2019) seems to be around<sup>2</sup>  $\dot{M} \approx 10^{-9} \dot{M}_{\text{Edd}}$ , it would even be possible to match the apparent blackbody temperature with the Sun’s temperature  $T \approx 6000$  K (Schnittman 2019).

In this paper, we wish to preliminarily examine, presumably for the first time, the effects that general relativity may directly have on the habitability of such putative Earth-like worlds through the long-term de Sitter and Lense-Thirring precessions of their spin axis impacting, among other things, their obliquity  $\varepsilon$ . Let us recall that it is defined as the angle between the planet’s proper spin angular momentum  $\mathbf{S}$  and the orbital angular momentum  $\mathbf{L}$  (Barnes et al. 2016).

The axial tilt of a planet is a key parameter for the insolation received from the orbited source of electromagnetic waves and, thus, its ability to sustain life over the eons (Laskar, Joutel & Robutel 1993; Williams & Kasting 1997; Laskar et al. 2004; Armstrong et al. 2014; Linsenmeier, Pascale & Lucarini 2015; Quarles et al. 2019; Kilic, Raible & Stocker 2017;

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<sup>1</sup> $G$  is the Newtonian gravitational constant, and  $c$  is the speed of light in vacuum. The gravitational radius  $r_g$  used by Schnittman (2019) as a unit of length is half the Schwarzschild radius.

<sup>2</sup>Here,  $\dot{M}_{\text{Edd}}$  is the Eddington accretion rate (Rezzolla & Zanotti 2013).

Shan & Li 2018; Quarles, Li & Lissauer 2019). Indeed, variations in obliquity  $\Delta\varepsilon(t)$  drive changes in planetary climate. If  $\Delta\varepsilon(t)$  are rapid and/or large, the resulting climate shifts can be commensurately severe, as pointed out by, e.g., Armstrong, Leovy & Quinn (2004). In the case of the Earth, its obliquity  $\varepsilon_{\oplus}$  changes slowly with time from  $\approx 22.1$  deg to 24.5 deg, undergoing an oscillation cycle with  $\Delta\varepsilon_{\oplus} \lesssim 2.4$  deg in about 41,000 yr (Quarles, Li & Lissauer 2019). The value of the Earth’s obliquity impacts the seasonal cycles, and its long-term variation affects the terrestrial climate (Milankovitch 1941), as deduced from geologic records (Kerr 1987; Mitrovica & Forte 1995; Pais et al. 1999). Such a moderate and benign modulation of  $\varepsilon_{\oplus}$  is sensibly due to the Moon (Laskar, Joutel & Robutel 1993), where the change would have been larger for a Moonless Earth (Laskar, Joutel & Robutel 1993; Lissauer, Barnes & Chambers 2012; Li & Batygin 2014). On the other hand, the obliquity of Mars  $\varepsilon_{\mars}$ , whose axis is not strongly stabilized by its tiny moons, experienced variations which have reached  $\Delta\varepsilon_{\mars} \approx 60$  deg (Ward 1973; Touma & Wisdom 1993). They contributed to the martian atmospheric collapse (Head et al. 2003, 2005; Forget et al. 2013), in addition to processes that altered the atmospheric pressure as well (Mansfield, Kite & Mischna 2018; Kite 2019). Venus, which, perhaps, was habitable in the early past (Way et al. 2016), may have experienced unexpected long-term variability of up to  $\Delta\varepsilon_{\venus} \approx \pm 7$  deg for certain initial values of retrograde rotation, i.e. for  $\varepsilon_{\venus}^0 > 90$  deg. (Barnes et al. 2016).

We will show that, in fact, under certain circumstances, also the general relativistic spin precessions do have potentially an impact on the ability of a rogue planet orbiting a Kerr SMBH, assumed maximally spinning, to sustain life due to the induced variations of its obliquity. Clearly, also other physical torques of classical nature due to  $N$ -body interactions with the planet’s equatorial bulge might impact  $\varepsilon$ ; they depend on specific circumstances like the presence of one or more large moons, other planets, passing stars, a more or less pronounced equatorial flattening, etc. In any case, such potentially non-negligible Einsteinian effects occur simply because the planet moves in the deformed spacetime of the SMBH, irrespectively of any other particular details on its physical characteristics and the presence of other interacting bodies or not. As such, they should enter the overall budget of the possible phenomena, treated so far, affecting the habitability of a SMBH’s planet, even at in a regime of comparatively weak gravity, i.e. far from the event horizon itself. By extrapolation, this aspect should have been, a fortiori, taken into account also for the planets of Gargantua in the film *Interstellar*, being very close to the event horizon.

The paper is organized as follows. In Section 2, we numerically integrate the time-dependent spin evolution of a rogue planet orbiting a maximally spinning Kerr SMBH in the Sun-Earth type scenario envisaged by Schnittman (2019). Analytical calculation of the de Sitter and Lense-Thirring spin precessions without any restrictions on either the orbital configuration of the gyroscope and the space orientation of its spin axis are performed in Section 3. Section 4 summarizes our findings, and offers our conclusions.

## 2. The general relativistic shift of the obliquity of the planetary spin axis

Here, for the sake of definiteness, we will consider in detail the situation envisaged in Schnittman (2019) consisting of a putative (Earth-like) planet orbiting a SMBH with  $M_{\bullet} = 1 \times 10^8 M_{\odot}$  at a distance  $r = 100 R_s$  from it in such a way that the apparent size of the lensed accretion disk is that of the Sun as viewed from the Earth. Although it is not strictly necessary from a mere computational point, in order to give greater weight to the considered scenario, it is tacitly assumed that the mass accretion rate of the SMBH is suitably tuned in such a way that the apparent blackbody temperature of the accretion disk is equal to the solar one.

In a rectangular Cartesian coordinate system centered at the SMBH, we simultaneously integrated the post-Newtonian<sup>3</sup> (pN) equations of motion of the planet (Poisson & Will 2014) along with the pN evolutionary equations of the planet’s spin axis (Poisson & Will 2014)

$$\frac{d\hat{\mathbf{S}}}{dt} = \boldsymbol{\Omega} \times \hat{\mathbf{S}}. \quad (1)$$

In Equation (1), the angular velocity vector  $\boldsymbol{\Omega}$  is the sum of the pN gravitoelectric de Sitter (dS) and gravitomagnetic Lense-Thirring (LT) terms

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\text{dS}} + \boldsymbol{\Omega}_{\text{LT}}, \quad (2)$$

with (Poisson & Will 2014)

$$\boldsymbol{\Omega}_{\text{dS}} = \frac{3GM_{\bullet}}{2c^2 r^2} \hat{\mathbf{r}} \times \mathbf{v}, \quad (3)$$

$$\boldsymbol{\Omega}_{\text{LT}} = \frac{G\mathbf{J}^{\bullet}}{2c^2 r^3} \left[ 3(\hat{\mathbf{J}}^{\bullet} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \hat{\mathbf{J}}^{\bullet} \right]. \quad (4)$$

In Eqs. (3) to (4),  $\mathbf{r}$ ,  $\mathbf{v}$  are the planet’s position and velocity, respectively, with respect to the SMBH, assumed maximally rotating. In the numerical integration, the “ecliptic” plane, i.e. the orbital plane itself, was assumed as reference  $\{x, y\}$  plane so that the inclination is  $I = 0$ .

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<sup>3</sup>Such an approximation is adequate in the present case since  $v^2/c^2 \simeq 5 \times 10^{-3}$ , and the largest pN acceleration experienced by the planet, i.e. the 1pN gravitoelectric one, is just  $\simeq 1\%$  of the Newtonian monopole. We included also the smaller 1pN spin and quadrupole, and the 2pN gravitoelectric accelerations in our dynamical model.

Furthermore, the spin axes of the planet and of the SMBH  $\hat{\mathbf{S}}$ ,  $\hat{\mathbf{J}}^\bullet$  were parameterized as

$$\hat{S}_x = \sin \varepsilon \cos \phi, \quad (5)$$

$$\hat{S}_y = \sin \varepsilon \sin \phi, \quad (6)$$

$$\hat{S}_z = \cos \varepsilon, \quad (7)$$

$$\hat{J}_x^\bullet = \sin \eta_\bullet \cos \varphi_\bullet, \quad (8)$$

$$\hat{J}_y^\bullet = \sin \eta_\bullet \sin \varphi_\bullet, \quad (9)$$

$$\hat{J}_z^\bullet = \cos \eta_\bullet, \quad (10)$$

where  $\varepsilon$ ,  $\eta_\bullet$  are the obliquities of  $\hat{\mathbf{S}}$ ,  $\hat{\mathbf{J}}^\bullet$ , respectively, to the orbital plane;  $\phi$ ,  $\varphi_\bullet$  are their azimuthal angles. Fixed generic initial conditions were adopted for the planet's orbital motion around the SMBH, assumed initially circular<sup>4</sup> ( $e_0 = 0$ ), while the initial orientations of the spin axes were varied from one run to another spanning a time interval  $\Delta t = 400$  yr long. Figure 1 depicts our results for the change  $\Delta \varepsilon(t)$  of the planet's obliquity experienced with respect to its initial value  $\varepsilon_0$ .

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<sup>4</sup>Here,  $e_0$  denotes the initial value of the orbital eccentricity  $e$ .

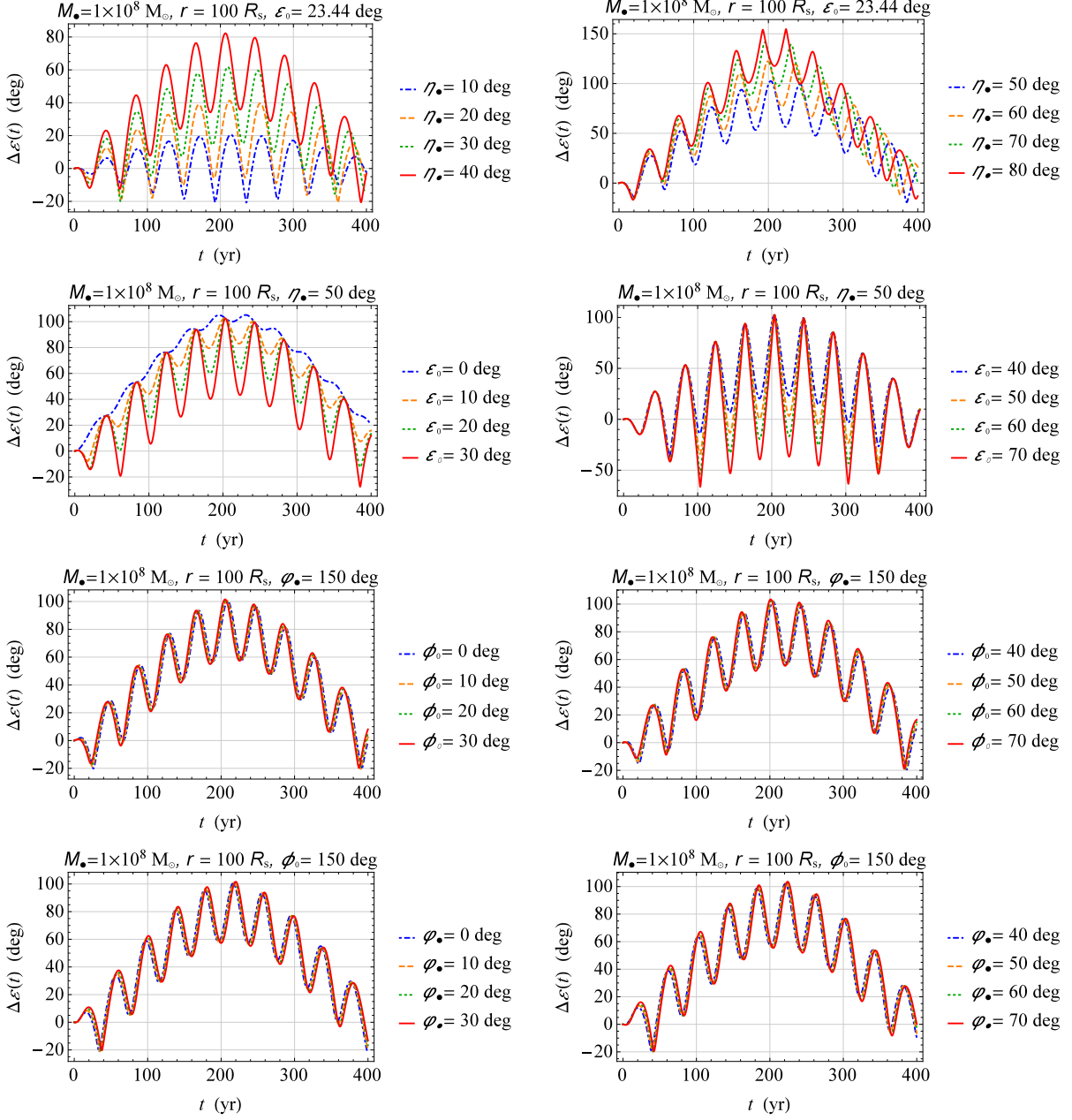


Fig. 1.— Numerically integrated time series of the shift  $\Delta\varepsilon(t)$  of the obliquity  $\varepsilon$  of the planet’s spin axis to the orbital plane for different values of some parameters by assuming  $M_\bullet = 1 \times 10^8 M_\odot$ ,  $r = 100 R_s$ , and an initially circular orbit ( $e_0 = 0$ ) with generic initial conditions. First and second rows: sensitivity to the obliquities  $\eta_\bullet$ ,  $\varepsilon_0$  of the spin axes of the SMBH and of the planet, respectively, by assuming  $\varphi_\bullet = 150$  deg,  $\phi_0 = 150$  deg for their azimuthal angles. Third and fourth rows: sensitivity to the azimuthal angles  $\varphi_\bullet$ ,  $\phi_0$  of the spin axes of the SMBH and of the planet, respectively, by assuming  $\eta_\bullet = 50$  deg,  $\varepsilon_0 = 23.4$  deg for their obliquities.

From the first row of Figure 1, it can be noted that the numerically integrated shift  $\Delta\varepsilon(t)$  of the planet’s spin obliquity, obtained by starting from an initial value  $\varepsilon_0 = 23.4$  deg identical to that of the Earth, exhibits rapidly varying changes, and depends strongly on the obliquity of the SMBH’s spin axis  $\eta_\bullet$ . Even for a small inclination of the orbital plane to the SMBH’s equator ( $\eta_\bullet \simeq 10 - 20$  deg), the resulting variation of the tilting of the planet’s equator can reach values as large as  $|\Delta\varepsilon| \lesssim 20 - 40$  deg. They can even become as large as  $|\Delta\varepsilon| \lesssim 80 - 150$  deg for  $\eta_\bullet$  approaching  $\simeq 90$  deg. The second row of Figure 1 shows that also the initial value  $\varepsilon_0$  of the tilt  $\varepsilon(t)$  of the planet’s equator to the orbital plane has a certain influence on its time evolution, although less marked than  $\eta_\bullet$ . It can be noted by comparing the dash-dotted blue curve in the upper right panel of Figure 1, corresponding to  $\varepsilon_0 = 23.4$  deg,  $\eta_\bullet = 50$  deg, to the somewhat different  $\varepsilon_0$ -dependent curves in the second row of Figure 1 obtained for  $\eta_\bullet = 50$  deg. According to the third and fourth rows of Figure 1,  $\Delta\varepsilon(t)$  is substantially insensitive to the azimuthal angles  $\phi_0, \varphi_\bullet$  of the spin axes of both the planet and the SMBH.

Figure 2 depicts the spatial path of the planet’s spin axis of one of the numerical integrations of the upper right panel of Figure 1.

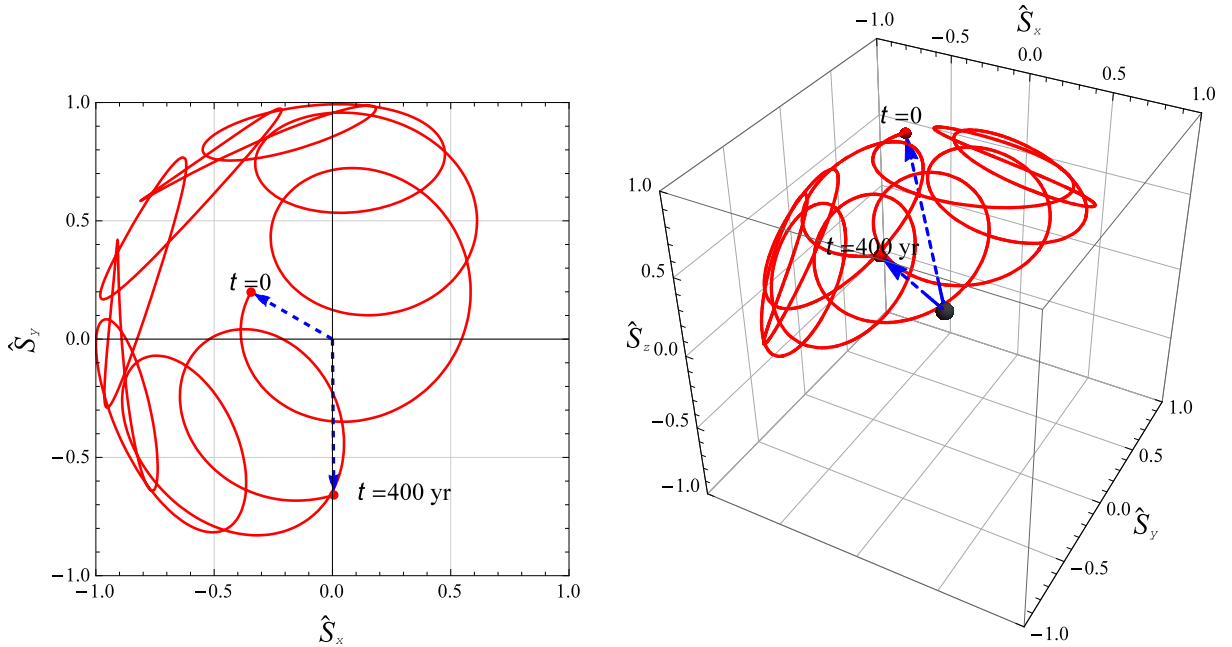


Fig. 2.— Left panel: numerically integrated hodograph, in red, of the planet’s spin axis projection, represented by the dashed blue arrow, onto the  $\{\hat{S}_x, \hat{S}_y\}$  plane over  $\Delta t = 400$  yr. Right panel: numerically integrated hodograph, in red, of the planet’s spin axis (dashed blue arrow) over the same time span. In both cases, a maximally rotating SMBH with  $M_\bullet = 1 \times 10^8 M_\odot$ ,  $\eta_\bullet = 50$  deg,  $\varphi_\bullet = 150$  deg is assumed. The initial conditions of the spin axis of the planet, which orbits at  $r = 100 R_s$  from the SMBH, are  $\varepsilon_0 = 23.4$  deg,  $\phi_0 = 150$  deg. Cfr. with the dash-dotted blue curve in the upper right panel of Figure 1.



Some qualitative features of the characteristic patterns of the time series displayed in Figure 1 can be inferred from an analytical calculation, which will be developed in the next Section.

### 3. Analytical calculation

Here, we will obtain general analytical expressions for the long-term, i.e. averaged over one orbital revolution, de Sitter and Lense-Thirring precessions of the obliquity  $\varepsilon$  and the azimuthal angle  $\phi$  of the spin axis  $\hat{\mathbf{S}}$  of a gyroscope orbiting a massive, spinning primary.

By evaluating the right-hand sides of Equation (1) for  $\mathbf{\Omega} = \mathbf{\Omega}_{\text{dS}}$ , as given by Equation (3), onto a Keplerian ellipse, assumed as unperturbed reference orbit, it is possible to obtain for the gravitoelectric de Sitter spin precession

$$\frac{d\hat{S}_x}{dt} = -\frac{3 n_b \mu (\hat{S}_y \cos I + \hat{S}_z \sin I \cos \Omega)}{2 c^2 a (1 - e^2)}, \quad (11)$$

$$\frac{d\hat{S}_y}{dt} = -\frac{3 n_b \mu (-\hat{S}_x \cos I + \hat{S}_z \sin I \sin \Omega)}{2 c^2 a (1 - e^2)}, \quad (12)$$

$$\frac{d\hat{S}_z}{dt} = \frac{3 n_b \mu \sin I (\hat{S}_x \cos \Omega + \hat{S}_y \sin \Omega)}{2 c^2 a (1 - e^2)}, \quad (13)$$

where  $\hat{S}_x, \hat{S}_y, \hat{S}_z$  are the components of the spin axis  $\hat{\mathbf{S}}$  of the orbiting gyro, assumed constant during an orbital revolution,  $a, e$  are the orbital semimajor axis and eccentricity, respectively,  $\mu \doteq GM$  is the gravitational parameter of the primary whose mass is  $M$ ,  $n_b \doteq \sqrt{\mu/a^3}$  is the Keplerian orbital mean motion,  $I$  is the inclination of the orbital plane to the reference  $\{x, y\}$  plane adopted, and  $\Omega$  is the longitude of the ascending node of the orbital plane, not to be confused with the modulus  $\Omega = |\mathbf{\Omega}|$  of the precessional angular velocity of the spin. The gravitoelectric averaged precessions of Eqs. (11) to (13) can be expressed in compact form as

$$\frac{d\hat{\mathbf{S}}}{dt} = \frac{3 n_b \mu}{2 c^2 a (1 - e^2)} \hat{\mathbf{k}} \times \hat{\mathbf{S}}, \quad (14)$$

where

$$\hat{\mathbf{k}} = \{\sin I \sin \Omega, -\sin I \cos \Omega, \cos I\} \quad (15)$$

is the unit vector directed perpendicularly to the gyro's orbital plane along the orbital angular momentum. The vectorial expression of Equation (14) agrees with Equation (10.146a) of Poisson & Will (2014) for  $e \rightarrow 0$ .

By taking the average of Equation (1) with  $\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\text{LT}}$ , given by Equation (4), over one gyroscope's orbital period one gets

$$\begin{aligned} \frac{d\hat{S}_x}{dt} = & -\frac{GJ}{8c^2a^3(1-e^2)^{3/2}} \left[ -(\hat{J}_y\hat{S}_z + 2\hat{S}_y\hat{J}_z)(1+3\cos 2I) + \right. \\ & + 6\hat{J}_y\hat{S}_z\cos 2\Omega\sin^2 I + 6(\hat{S}_y\hat{J}_y - \hat{S}_z\hat{J}_z)\cos\Omega\sin 2I - 6\hat{J}_x\hat{S}_y\sin 2I\sin\Omega - \\ & \left. - 6\hat{J}_x\hat{S}_z\sin^2 I\sin 2\Omega \right], \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{d\hat{S}_y}{dt} = & -\frac{GJ}{8c^2a^3(1-e^2)^{3/2}} \left[ (\hat{J}_x\hat{S}_z + 2\hat{S}_x\hat{J}_z)(1+3\cos 2I) + 6\hat{J}_x\hat{S}_z\cos 2\Omega\sin^2 I - \right. \\ & \left. - 6\hat{S}_x\hat{J}_y\cos\Omega\sin 2I + 6(\hat{S}_x\hat{J}_x - \hat{S}_z\hat{J}_z)\sin 2I\sin\Omega + 6\hat{J}_y\hat{S}_z\sin^2 I\sin 2\Omega \right], \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{d\hat{S}_z}{dt} = & -\frac{GJ}{8c^2a^3(1-e^2)^{3/2}} \left[ -\hat{J}_x\hat{S}_y + \hat{S}_x\hat{J}_y + (-3\hat{J}_x\hat{S}_y + 3\hat{S}_x\hat{J}_y)\cos 2I - \right. \\ & \left. - 6(\hat{J}_x\hat{S}_y + \hat{S}_x\hat{J}_y)\cos 2\Omega\sin^2 I + 6\hat{J}_z\sin 2I(\hat{S}_x\cos\Omega + \hat{S}_y\sin\Omega) + \right. \\ & \left. + 6(\hat{S}_x\hat{J}_x - \hat{S}_y\hat{J}_y)\sin^2 I\sin 2\Omega \right], \end{aligned} \quad (18)$$

where  $\hat{J}_x, \hat{J}_y, \hat{J}_z$  are the components of the spin axis  $\hat{\boldsymbol{J}}$  of the primary; also in this case,  $\hat{S}_x, \hat{S}_y, \hat{S}_z$  were kept constant during the orbital averaging. The gravitomagnetic averaged precessions of Eqs. (16) to (18) can be cast in the following vectorial form

$$\frac{d\hat{\boldsymbol{S}}}{dt} = \frac{GJ}{2c^2a^3(1-e^2)^{3/2}} \left\{ 3 \left[ (\hat{\boldsymbol{J}} \cdot \hat{\boldsymbol{l}}) \hat{\boldsymbol{l}} + (\hat{\boldsymbol{J}} \cdot \hat{\boldsymbol{m}}) \hat{\boldsymbol{m}} \right] - 2\hat{\boldsymbol{J}} \right\} \times \hat{\boldsymbol{S}}, \quad (19)$$

where

$$\hat{\boldsymbol{l}} = \{\cos\Omega, \sin\Omega, 0\} \quad (20)$$

is a unit vector directed along the line of the nodes toward the ascending node, and

$$\hat{\boldsymbol{m}} = \{-\cos I \sin\Omega, \cos I \cos\Omega, \sin I\} \quad (21)$$

is a unit vector lying in the gyro's orbital plane directed transversely to  $\hat{\boldsymbol{l}}$  such that the basis  $\hat{\boldsymbol{l}}, \hat{\boldsymbol{m}}, \hat{\boldsymbol{k}}$  is right-handed. It should be noted that Equation (19) agrees with Equation (10.146b) of Poisson & Will (2014) for  $\hat{J}_y = \hat{J}_y = 0, \hat{J}_z = 1$  and  $e \rightarrow 0$ .

By adopting the orbital plane as reference  $\{x, y\}$  plane, i.e.  $I = 0$ , and the parametrization of Eqs. (5) to (7) for the spin axes of the gyroscope and the primary, we have

$$\dot{\varepsilon} = -\frac{1}{\sin \varepsilon} \frac{d\hat{S}_z}{dt}, \quad (22)$$

$$(\dot{\phi})^2 = \frac{1}{\sin^2 \varepsilon} \left[ \left( \frac{d\hat{S}_x}{dt} \right)^2 + \left( \frac{d\hat{S}_y}{dt} \right)^2 - \cot^2 \varepsilon \left( \frac{d\hat{S}_z}{dt} \right)^2 \right], \quad (23)$$

where the expressions for  $d\hat{S}_x/dt$ ,  $d\hat{S}_y/dt$ ,  $d\hat{S}_z/dt$  to be inserted in Eqs. (22) to (23) are Eqs. (11) to (13) or Eqs. (16) to (18) written in terms of Eqs. (5) to (10). It turns out that the obliquity  $\varepsilon$  is not affected by the de Sitter precession which, instead, changes the azimuthal angle  $\phi$ . Instead, the Lense-Thirring field impacts both  $\varepsilon$  and  $\phi$ . By specializing our results to the SMBH-planet scenario considered here, the pN rates are, thus

$$\dot{\varepsilon}_{\text{dS}} = 0, \quad (24)$$

$$\dot{\phi}_{\text{dS}} = \frac{3 n_b \mu_\bullet}{2 c^2 a (1 - e^2)}, \quad (25)$$

$$\dot{\varepsilon}_{\text{LT}} = \frac{G J^\bullet \sin \eta_\bullet \sin(\varphi_\bullet - \phi)}{2 c^2 a^3 (1 - e^2)^{3/2}}, \quad (26)$$

$$\dot{\phi}_{\text{LT}} = \frac{G J^\bullet [2 \cos \eta_\bullet + \cos(\varphi_\bullet - \phi) \cot \varepsilon \sin \eta_\bullet]}{2 c^2 a^3 (1 - e^2)^{3/2}}. \quad (27)$$

According to Equation (26), the precession of the obliquity, entirely gravitomagnetic in nature, vanishes if the spin of the SMBH is perpendicular to the gyro's orbital plane, independently of any other parameters. Such a feature is confirmed by a dedicated set of numerical integrations in which the initial conditions of  $\phi$ ,  $\varepsilon$  and the value of  $\varphi_\bullet$  were varied by keeping  $\eta_\bullet = 0$ . In all other cases, the spin-induced gravitomagnetic field of the SMBH is crucial for the effects of interest here not only because it directly induces the change of the obliquity, but also because it indirectly characterizes the resulting nontrivial temporal patterns exhibited in Figure 1. Indeed, the azimuthal angle  $\phi$  entering Equation (26) is slowly time-dependent because of both Equation (25) and Equation (27) as well. If, ad absurdum, the variation of  $\phi$  were only due to Equation (25), it would be easy to analytically model its variation as a simple linear trend in  $t$ , insert it into Equation (26), and analytically integrate it by obtaining a harmonic signature. Such a feature is confirmed also by a numerical integration performed by switching off the Lense-Thirring acceleration in the planet's equations of motion, but keeping the gravitomagnetic evolution equation for its spin. Actually, both  $\varepsilon$  and  $\phi$  do change simultaneously because of the combined action of the coupled

Equation (25) and Equation (27), and of the pN gravitoelectromagnetic equations of motion of the planet themselves. Note also that the right-hand-side of Equation (27) contain both  $\phi$  itself and  $\varepsilon$  as well. Even if one were to choose some suitable initial values like  $\varepsilon_0 = 90$  deg or  $\varphi_\bullet - \phi_0 = 90$  deg, such conditions would not be kept by the subsequent temporal evolution of  $\phi(t)$  and  $\varepsilon(t)$ , as confirmed by dedicated numerical integrations. Thus, in the general case, it is not possible to put any simple analytical expression of  $\phi(t)$  in Equation (26). This explains, at least qualitatively, the nature of the numerically integrated patterns displayed in Figure 1, and elucidate that they are essentially gravitomagnetic in nature.

#### 4. Summary and conclusions

Recently, the possibility that a huge number of rogue telluric planets, potentially able to sustain Earth-like life under certain circumstances, may form in the neighbourhood of SMBHs has gained growing attention in the literature. Several physical phenomena of different nature have been considered so far in connection to their role in favouring or disadvantaging the emergence of habitable ecosystems in such worlds which would receive the required energy input mainly from a possible accretion disk surrounding the SMBH. Also science fiction has staged such a possibility in the famous film *Interstellar*. Among the various scenarios appeared in the literature, an interesting one consists of a putative planet orbiting a SMBH at such a distance  $r$  that its lensed disk would appear to an observer on the planet orbiting just outside the SMBH’s equatorial plane as large as the Sun as seen from the Earth; for  $M_\bullet = 1 \times 10^8 M_\odot$ , it would be  $r = 100 R_s$ . For a certain mass accretion rate, it would even be possible to set the disk’s temperature equal to the solar one.

Here, we demonstrated that, among the many pros and cons that must be weighed in a plausible assessment of the habitability of such a world, there is also the effect that general relativity exerts on the tilt  $\varepsilon$  of its spin axis to the “ecliptical” plane. Indeed, by numerically integrating the planet’s pN equations of motion along with the pN evolution equations of its spin axis, it turned out that its obliquity  $\varepsilon$  may experience remarkable changes  $\Delta\varepsilon(t)$  over a comparatively short time, mainly depending on the obliquity  $\eta_\bullet$  of the spin axis of the SMBH, assumed maximally rotating, and, although to a lesser extent, on the initial value  $\varepsilon_0$  of the planet’s obliquity itself. Indeed,  $\Delta\varepsilon(t)$  undergoes oscillating variations over a time span, say,  $\Delta t = 400$  yr whose size may be as large as tens or even hundreds of degrees. The largest effects occur for  $\eta_\bullet$  approaching 90 deg, but also if the SMBH’s spin axis is nearly perpendicular to the orbital plane ( $\eta_\bullet \simeq 10 - 20$  deg) the amplitude of the change of the planet’s obliquity can reach the  $\simeq 20 - 40$  deg level. An analytical analysis demonstrated that such an effect is essentially due to the spin-induced gravitomagnetic field of the SMBH.

It is straightforward to infer that, a fortiori, the general relativistic effects on the spin axes of the fictional planets of the film *Interstellar*, much closer to their SMBH than the case discussed here, should be much more pronounced, thus notably impacting the pattern of the sky as seen

from them. Such a feature was apparently overlooked both in the movie and in its semipopular scientific interpretations that have appeared so far.

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