The Structure of Reality, or Where to Find the Final Theory?

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The main objective of the present article is the methodological analysis of the structures of physical theories that could apply to the theory of quantum gravitation or the unified theory of all interactions (other terms are the "Final Theory" or "the Theory of Everything", TOE).

In the first step of this discussion, it is shown that, unlike widely believed, quantum theory, in principle, allows representation by local classical hidden variables. The possibility of such representation is proved by the possibility of an exhaustive simulation of quantum systems by means of a local and classical device — the computer equipment plays a role of the local classical hidden variables. Details regarding the realization of such a representation are discussed using the example of an actual computer program simulating a correlation experiment using Eeinstein-Podolsky-Rozen pairs. This example explicitly violates the theorems of impossibility for hidden classical variables in quantum theory.

The fact that these theorems ignore the possibility for a situation, which, in this article, is characterized as the splitting of the layers of reality, is the reason that a possible violation of the theorem of impossibility for hidden variables exists. There are two such layers in the computer example — a reality layer where computer equipment exists, and a layer for simulated quantum reality. The analysis of this example results in a general idea about the layers of reality, which are the main subject of the subsequent discussion.

The computer example plays a role of the existence theorem. It follows that, in principle, a fundamental local and classical structure may exist behind physical quantum reality. However, such a "local realism" leads to the idea that an immense "space container" exists for classical objects of such a layer of reality. The problem is overcome if the fundamental ontology is classical, but nonlocal. Then, a space container for it is not required. It is shown that such a classical, but nonlocal, structure is very similar to a formal mathematical system. It leads to a thought that the ideal mathematical system can be a fundamental ontology of the TOE, or it is reminiscent of something beyond mathematics — a nonreducible pseudomathematical structure.

In this regard, the analysis of the nature of mathematics is given. It is shown that mathematics are not only a result of the imagination of people, but that mathematical forms and all mathematics holistically exist objectively. Moreover, the statement about the objective existence of mathematical forms has an empirical status based on Popper's criterion of falsifiability. It transfers a question of objectivity for mathematics from the field of philosophy to the field of empirical science. Then, a connection on the bases of mathematics and physics is established — namely, with the existence of the classical sector of quantum theory and with a causal structure of space-time. In this sense, the existing mathematics are not only objective, but they also possess physics. That is, mathematics, in a sense, is a thing but it is not a thought, and this quality can be considered as an objective layer of reality that can be a substrate of the physical world.

In the final part of this paper, several modern directions in the quantum gravity theory or TOE (string theory, loop quantum gravity, and causal sets) are considered regarding, as far as these theories *are concerned, the formation of an abstract mathematical substratum. It is shown that the tendency to develop a of structure like an abstract mathematical substratum definitely exists. This means that a TOE can be not just a physical theory, but rather an abstract mathematical structure.*

Keywords: quantum gravity, Final Theory, Theory of Everything, local realism, mathematical Platonism, basic ontology, emergence

Introduction

The last word in physics comes from two physical theories that have been extremely successful so far — quantum mechanics and general relativity theory. Quantum mechanics, together with its relativistic form, the quantum field theory, has enabled the understanding of microphysics, which includes atomic and nuclear physics, physics of the condensed state, and elementary particle physics. Recently, it is possible to note the rapid development of quantum informatics including quantum cryptography and quantum computing. Discovery of the Higgs boson, which was predicted by quantum field theory, became the triumph of the so-called Standard Model of particle physics [ATLAS, 2012; CMC, 2012]. General relativity gave the exact prediction for the strength of the bending of light in a gravitational field and the relativistic perturbations in celestial motions, also some fine effects predicted for the precession of rotating bodies in gravitational fields were confirmed. The effects of general relativity also plays a prime role in satellite navigation systems. A recent triumph for general relativity, on an event scale comparable to the discovery of the Higgs boson, came from a direct observation of gravity waves from the merging of two black holes [Abbot, 2016]. Use of the general relativity, together with quantum mechanics, recently allowed the development of modern relativistic astrophysics, which has been extremely successful. The quantitative cosmology and the so-called Standard Cosmological Model have been developed. The Standard Cosmological Model contains only six tuned parameters, but it also describes, very precisely, a wide set of the phenomena — fine features of the anisotropy of the cosmological microwave background, a distribution of matter in the Metagalaxy, etc. The physics of such relativistic astrophysical objects, such as neutron stars and black holes, are substantially understood. Along with numerous indirect pieces of evidence for the existence of black holes, the mentioned above observation of the gravity waves from merging black holes [Abbot, 2016] was a new confirmation for the existence of black holes.

At the same time, quantum theory and general relativity have absolutely different conceptual bases and are fundamentally incompatible with each other. The formulation of quantum theory demands a smooth static background, which role is played by a flat Minkowski space in quantum theory. On the contrary, the dynamical nature of space-time geometry and the idea of an absence of static background in the theory are the cornerstones for general relativity — general relativity is the so-called background-independent theory (unlike quantum mechanics). At the same time, general relativity is, in essence, a classical theory in the sense that classical behavior is characteristic of all the observables — they, in principle, can be measured with any accuracy without their perturbation. There are two important issues here. First, general relativity, being a classical theory, does not allow the calculation of gravitational fields for a quantum particle whose state is characterized not by coordinates and momentum, but by a wave function. The state of the gravitational field for a quantum particle must be described in a quantum way that general relativity, as a classical theory, does not allow. Therefore, general relativity must be quantized. Secondly, quantum mechanics says that any dynamical system must be quantized. In general relativity, space and time are dynamical fields; therefore, the smooth behavior of space-time must be replaced, at the microscale, with a picture of chaotic quantum fluctuations or another quantum description. It follows that the static space background, on which the quantum theory is formulated, is only a low-energy approximation, and at the fundamental level, it is simply absent. This means that quantum mechanics loses its conceptual basis and must be considerably reformulated.

Despite these contradictions, the nature is uniform; therefore, both quantum theory and general relativity are "actually" somehow coordinated among themselves in nature. It is necessary to guess how it is arranged. Modification of both theories or a reduction of both to some common basis should be possible. The required theory has to describe, in a unified manner, both quantum space-time and all quantum fields, which is frequently called the "Theory Of Everything," the "TOE," or the "Final Theory." The last term means that all other levels of existence for matter, in principle, must be deduced as special solutions of the TOE.

TOE must be a quantum theory of gravitation that must also include the quantum theory for all types of particles and interactions. The search for a quantum theory of gravitation has an old history, which we will not be elaborated upon here, and now this science includes a variety of directions. Among those that are considered, at the subjective view of the author, to be predominant, include: string theory [Zwiebach, 2009]; loop quantum gravity in two forms — as result of a canonical quantization of general relativity [Rovelli, 1998] and as the theory of "spin foam" [Baez, 1998] based on the Feynman path integral; casual sets theory [Rideout, 1999]; several forms of the theory of dynamic triangulation [Ambjorn, 1995; Ambjorn 2001]; Regge calculus [Ponzano, 1968; Bittner, 2003]; and several forms of "toy models" such as "quantum graffiti" [Konopka, 2006]; and many others. It should also be noted the one of the first historical approaches to quantum gravity is known as quantum geometrodynamics, which was proposed by Wheeler-DeWitt [DeWitt, 1967]. This direction is now used generally in the simplified quantum cosmological models based on the so-called mini-superspaces, but the most important ideas that appeared in it were included into more modern constructions. An example is loop quantum gravity.

Many theories have led to many interesting and deep results, but there is still not yet a self-consistent quantum theory of gravity despite enormous efforts spent searching for it within the last several decades. It seems that various theories grasp separate details of the true TOE, but in general, the advances in the direction of the Final Theory are surprisingly sluggish. It is difficult to move past the impression that an intellectual jump, which will change the available understanding, is necessary for overcoming the difficulties that occurred during the creation of the special, and then, the general relativity theory or quantum mechanics. Perhaps there is a lack of a new fundamental principle that can reveal the limitations of the used approaches.

In the present paper, a methodological analysis is carried out, which shows that the problem stems from the modern notion of what is a physical theory at the most fundamental level. This notion may be somewhat limited. Discussion begins with one special physical problem — the problem of correlating the measurements of the spins of entangled Einstein-Podolsky-Rozen pairs of particles (EPR pairs), and Bell's theorem combined with the problem of hidden variables in quantum theory. A counterexample to the Bell's theorem is construcyed, which leads to some interesting new concepts, and, at the same time, does not use of any essential hypotheses or speculation. However, the attentive analysis of these results lead to a range of hypotheses and opportunities for a significantly more speculative nature, but, at the same time, they have a direct bearing on the possible nature of quantum gravity or the Final Theory.

Violation of Bell's theorem in quantum virtual reality

Let us begin with a short reminder of the correlate experiment with EPR-pairs of spin and the related Bell's theorem [Grib, 1984; Bell, 1966]. When the initial states of pairs for quantum particles *A* and *B*, with spins of 1/2, are prepared so that the particle spins are directed precisely to opposite directions, then the complete spin of the particle pair is equal to zero. Nothing else about the initial state of the system is known, such as a state called the spin singlet. After the preparation of the initial state, the particles can be divorced over an arbitrarily long distance. The experiment consists of measuring the projections of spin for particles *A* and *B* in various directions. For each particle, each measurement may have only two possible outcomes: either the spin is directed in the chosen direction, or the spin is directed against it (because the spin of a particle is equal to 1/2, while for greater values of spin, the number of outcomes would be greater). If the spin is directed in the chosen direction, then the value of +1 is the measurement result; if the spin is against the chosen direction, then a value of –1 occurs. Correlations of the observed results for measuring the spins of particles *A* and *B* in various directions are defined. In this experiment, these correlations represent mean values of the products with observed values of +1 or –1.

Let two frames, (*X, Y, Z*) and (*X', Y', Z'*), represent the measurement of spin projections, while frame (*X', Y', Z'*) concerns system (*X, Y, Z*) when angle Q is counterclockwise around the *Y* axis, and the directions of the *Y* and *Y'* axes coincide with one another. For definiteness, we assume that the system (*X, Y, Z*) is related to the device measuring the spin of particle *A*, and that the system (*X', Y', Z'*) is related to the device measuring the spin for particle *B*. Let S_Z^A be the result of measuring the spin of a particle *A* along the *Z* direction, etc. Then, Bell's theorem states that, if the measurement results are determined *by local hidden variables*, the following inequality must be satisfied, where the angular brackets denote averaging (or the correlation of the measurement results).

$$|C| = |\langle S_Z^A S_{Z'}^B \rangle + \langle S_X^A S_{X'}^B \rangle + \langle S_X^A S_{Z'}^B \rangle - \langle S_Z^A S_{X'}^B \rangle| \leq 2 \qquad (1)$$

Quantum mechanics, however, predicts the following result for quantity *C* in Equation (1):

$$C(\Theta) = 2\sqrt{2}\,\sin(\Theta - \pi/4) \qquad (2)$$

Obviously, at some Θ the absolute value of *C* (Θ) is greater than two and inequality (1) is violated. Experiments confirm the prediction made using quantum mechanics (2).

In the above formulations, the term "local hidden variables" was used. The introduction of local hidden variables is one of the ways to explain the statistical nature of quantum-mechanical measurements. It is assumed that, behind the wave function of a particle, in fact, lies a statistical distribution of some classical quantities, and that the random nature of the measurement results is determined by the randomness of the sample from this distribution. In this case, the variables are local in the sense that the result of measuring the first spin state cannot, in any way, affect the measurement result for the second spin state. Indeed, if this were possible, then, for measuring moments of time separated by a spacelike interval, this would mean transferring information faster than the speed of light, which is fraught with a violation of causality. The presence of local hidden variables "behind the scenes" of quantum dynamics is sometimes called "local realism." From the mathematical point of view, the presence of local realism means that mean values, like those used in Bell's formula (1), can be represented in the following form [Grib, 1984]:

$$\langle S_a^A S_b^B \rangle = \int \rho(\lambda) A(a,\lambda)\, B(b,\lambda)\, d\lambda \qquad (3)$$

where l is a set of hidden variables appropriate for the system, r(l) is a probability distribution of the hidden variables described by the initial state (in other words, a common past of the particles), the parameters *a* and *b* describe the condition of the measurements on *A* and *B* respectively (the direction of the axes relative to the measured projection of spin), and functions *A* (*a*,l) and *B* (*b*,l) describe the measurements themselves. Locality means the function *A* (...,l) cannot depend on parameter *b*, and the function *B* (...,l) cannot depend on parameter *a*.

The fact that quantum mechanics, according to Equation (2), predicts the violation of Bell's inequality, which is confirmed through experimentation, is interpreted in such a way that local realism in quantum mechanics is excluded. Now, we will construct a meaningful counterexample, which shows that the interpretation of Bell's theorem is not so simple.

A good example of a system with local realism is a classical (not quantum) computer. A state of the computer is described by a set of states in its memory cells. Each memory cell can be in a state of 0 or 1, and these states, in principle, can always be measured without disturbances because of the classical nature of the computer. Moreover, measurement over one particular cell of memory does not depend on the measurements performed on any other cell, and it does not affect them. The state of the computer, at some point in time, completely determines its state during the next step of time; therefore, its evolution is completely classical and deterministic.

Now, however, it is not difficult to understand that the evolution of any *quantum* system, in principle, can be reduced to the operation of some *classical* computer.

All mathematical problems are divided into algorithmically solvable and algorithmically unsolvable ones. An example of an algorithmically solvable problem is the search for the greatest common divisor of two integers — it may be solved using the well-known Euclidean algorithm. An example of an algorithmically unsolvable problem is the solution to a general Diophantine equation (Hilbert's tenth problem) [Matiyasevich, 1993]. An algorithmically solvable problem can always be solved by a computer in its automatic mode.

The problem of computing quantum evolution, along with simulations to produce results for the quantum measurements, is algorithmically solvable. More precisely, quantum evolution can be described in an algorithmically solvable way with an arbitrarily high accuracy, which is equivalent to an exact computing. It is easy to understand why this happens.

Indeed, the states of quantum systems are represented by vectors in Hilbert space. A vector in Hilbert space is a well-understood object that can, in principle, be represented in a computer with any given accuracy, even if the Hilbert space has infinite dimensions. In principle, this situation is like the representation of ordinary continuous classical fields in a computer with the required specified accuracy. From a formal point of view, a vector in Hilbert space, for a computer, is simply a chain of complex numbers.

The evolution of systems in quantum theory is represented by unitary transformations of system states, such as vectors in Hilbert space. From a formal point of view, such a unitary transformation is either a multiplication of a column vector, which represents the state of the system, by a unitary matrix, which represents the transformation corresponding to the evolution, or the result of solving some linear differential equation with initial conditions. Nothing is impossible when carrying out such operations for a conventional classical computer.

Finally, the third and last component of quantum theory consists of measurements. The measurements are characterized by the probabilities of obtaining, as the output of a measurement procedure, certain values for the observed quantities, and the probabilities themselves are determined by the Born-von Neumann projection postulate. To calculate the probabilities, it is not necessary to calculate anything more complicated than a scalar product of the vectors in Hilbert space. Again, this is an algorithmic procedure that can be easily executed on a computer. These probabilities cannot only can be calculated, but, if necessary, the corresponding probabilistic behavior of the measurement outcome can be simulated using random number generators. Statistical properties of the random number generator can be made arbitrarily close to any predefined requirements (this situation is like the approximation of continuous fields and functions by discrete computer memory states).

Because simulating quantum evolution is algorithmically solvable, the computer program can force itself to behave exactly like any quantum system. Because the computer is a system with local realism, then we get that computer to play a role in the hidden local realism, or with the local hidden variables in relation to the simulated quantum reality. This is a counterexample of Bell's theorem.

The fact that the quantum reality of our example is virtual, and not real, does not cast a shadow over the constructed example. This is easy to understand if one imagines that simulated quantum reality is so rich that an observer is simulated within it. Then, an observer will have no reason to suspect that he is "not real." And yet, the deep ontology of his quantum world will be the local realism of the computer that simulates the observer's reality. Although, it is, in principle, impossible for an observer to be simulated on a classical computer (which will be discussed below), it is possible to simulate relatively uncomplicated measuring devices along with the quantum systems that they measure, and the behavior of such devices will not be defective in comparison to the behavior of real devices.

The constructed counterexample shows that some essential link in the arguments leading to the conclusion that local realism is impossible in quantum mechanics was omitted. What could it be? To understand this issue in detail, the author of this article wrote and debugged a simple computer program — a simulator measuring the spins of entangled EPR pairs. Such a program should accurately simulate quantum mechanical measurements, and at the same time, its structure should be so simple that it can be studied in detail so that it can be understood how quantum behavior can combine with the local realism of a computer. Some details of the program's implementation are given in our article [Panov, 2014].

A detailed analysis of the algorithm's operation showed that the correlators of the spin measurements are indeed described by an expression of type (3), where the role of parameter l is played by the value of the random number generator. However, the functions *A* (...,l) and *B* (...,l) are "incorrect:" parameter *b*, which characterizes the direction of the measurement axis over spin *B*, appears in function *A* (...,l), and vice versa. That is, we get the following picture. The functions *A* (...,l) and *B* (...,l), in the "computer reality," are represented by states of classical memory cells, and therefore, are expressed completely in terms of the local realism. However, from the point of view of "virtual reality" itself, the same functions reflect the nonlocal nature of quantum entanglement — the function *A* (...,l) is somehow "mystical" (from the virtual reality point of view) depending on the "remote" parameter b, and vice versa. Thus, the same reality of a computer with different points of view turns out to be different: it is local from the point of view of the computer, while it is not local from the point of view of the simulated virtual reality. There arises the relativity of locality to some different points of view, which are omitted in the initial analysis of the corollaries for Bell's theorem. This understanding gives an interesting correction to the usual analysis of Bell's theorem and explains the possibility of constructing counterexamples. For us, however, the very possibility of different points of view on locality is more important here. *What is the difference between these points of view?*

This is a new formulation of the question, and we postulate that these are views from "*different layers of reality.*" The form of the answer to the question asserts that such things as "different layers of reality," in general, can exist and be studied. In our concrete example, we have exactly two such layers: the reality layer of the computer's "hardware" and the layer of virtual reality simulated by it. The computer layer plays the role of a substratum, where a layer of virtual reality arises as an image in some emergent fashion. We have a substrate-image relationship between the layers of reality.

We have firmly stood with our feet on the basis of a real (computational) experiment, which can easily be realized (and was actually implemented) simply in a table. In the analysis, there was nothing but convenient new terminology. However, what we have done is, in fact, a proof of the existence theorem, which says that the introduced terminology, which relates different layers of reality, is meaningful and non-contradictory. However, we now turn to a somewhat more speculative generalization of this experience.

Quantum physics in the real world is based on the classical substrate

So far, we have only discussed two layers of reality: a layer of virtual reality from a computer program and a layer of "physical reality" from the computer hardware. However, in exact analogy and just as the quantum dynamics was simulated by a computer program, an emergent manifestation of the classical local realism of the computer-triggered circuits, and the quantum dynamics of the "physical level of reality" observed by us in experiments can, in principle, be an emergent manifestation of some deeper classical and local reality. The previously presented examples prove the logical possibility of such a situation. In other words, real quantum dynamics can have the meaning of an image constructed on some substrate that is hidden from us, which can possess the properties of both classicality and locality. Experimental observations for the violation of Bell's inequalities and the theorem of quantum mechanics, in regard to the impossibility of classical local hidden variables, do not shut down local realism on a deeper ontological level, because these theorems refer only to the reality of our "current" physical level of reality. They do not consider the possibility of the existence of hidden substratum layers.

What is the nature of the hidden reality substrate?

First, it should be noted that the examples discussed above, which associate the simulation of quantum dynamics in computer computational models, were only "stagings" that conveyed the simplest way to conclude that quantum dynamics can be realized as an image in a classically local substrate. In fact, there are other ways of constructing such an image. For example, the dynamics of a wave function, in time, can simply be written to an external classical information carrier, and this is completely local-classical and, moreover, the static record also adequately represents quantum dynamics, because the time simulation and this static record are in a one-to-one correspondence. In this case, not only will the quantum behavior of reality be emergent, but time will too. We can say that our sense of the current time, along with all the other dynamics, can be recorded in a static timeless substrate and is an illusion of the internal, "frog" point of view of an observer, which is unlike the external, "eagle" point of view. In accordance with the eagle's perspective, all the living, unfolding time in the world can be a static object. One can imagine that such a record is generated by the fixation of successive states of the computer, which are actually simulated quantum dynamics, but this is not necessary at all. The record can have an arbitrary origin and it is important only that the evolution of a quantum wave function can be represented by text on a classical storage carrier. Thus, there is a wide class of different possibilities for representing quantum dynamics as an image in a local-classical substrate. You can consider some mixture of both possibilities considered above, or, for a change, you can imagine, instead of a symbolic record, an analog record like recording sound on magnetic tape. Note that a computer simulation does not have to be digital; it is possible to imagine a simulation for quantum reality that is analog, but for a classic computer. All this does not change the essence of the discussion.

Although we concluded that a locally classical substratum for physical quantum dynamics is logically possible, but the real local-classical substratum of quantum theory, if one exists, does not have to resemble of a classical computer or a classical information record. The examples demonstrate only the logical possibility of a classically-local substrate, but nothing beyond this. Moreover, an analogy that is too straightforward compared with our examples have unpleasant consequences.

If we try to imagine the quantum world as a simulation in some classical automaton, or that it is a "static" record in a local-classical substrate, then we will have to assume that there exists some immense spatial container for such devices or information carriers. Indeed, even to simply record, with reasonable accuracy, the wave function of the electron shell of just one multielectron atom, e.g., the uranium atom, the volume of the entire visible universe would not suffice, even if one bit of information is placed in each Planck cell of space. The configuration space for 92 electrons from the uranium shell has a dimension of $92\times3\times2=552$ (for a nonrelativistic description with electron spin considered), and if we require at least 1000 nodes in the coordinate grid along each coordinate to write the wave function, then a memory of $1000^{552} = 10^{1656}$ complex numbers would be required. At the same time, in the visible part of the universe, there are only about 10^{153} Planck cells of space (each is 10^{-33} cm in size). Apparently, about 1,500 orders of magnitude are still missing. For this reason, real classical computers, although true, can simulate the dynamics of quantum systems without any damage, but there are very limited systems in terms of the complexity that could be modeled with exhaustive accuracy. For example, quantum computers can be simulated by classical computers, but only very small quantum computers that contain no more than about 30 qubits of quantum memory. For the same reason, it is very difficult to imagine a computer simulating a quantum world containing a thinking observer. One can, of course, imagine the very high dimensionality of the "spatial container" of the universe computer, but such steps seem somewhat artificial.

Thus, if a local-classical substrate of our quantum world exists, then it is unlikely that the locality in it could be understood in a primitive-spatial manner. A more interesting possibility is that the substrate can possess features of classicity, but not locality. In particular, it better resembles abstract information or a mathematical structure rather than a physical system located in some kind of space. In this case, the above unpleasant question about the "container" for the existence of such a monstrously complex structure is automatically removed, because the corresponding mode of existence has, in some sense, a completely non-spatial character, where it does not require "space." Objective existence is not identical with existence in space and time.

Mathematical and pseudomathematical substrates

Mathematics has some interesting features that relate it to a classical, but non-local system.

Some analogy of physical measurement or observation is, in mathematics, a calculation of the values of mathematical objects (this analogy will be discussed in additional detail below). Very remarkable is the fact that the calculation of the value of a mathematical object does not affect the results of subsequent calculations for the same object. That is, the calculations have an undisturbed character, such as with an ideal measurement of classical physics. This is the classicism of mathematics. The classicity of mathematics is by no means a banality as it is easy to give an example of a system resembling mathematics, but that does not possess the property of classicity. This example is given, for example, by the mental world of man. Like mathematics, it has an "ideal" nature, but let us try to follow the course of thought — this will immediately change the content of the thought process. The classicity of mathematics is our happiness, because only this property allows us to create stable mathematical models, which provide a definite contribution to the "unreasonable effectiveness of mathematics in the natural sciences" [Wigner, 1960].

The nonlocality of mathematics corresponds to the fact that mathematical objects do not have a space-time binding, nor do they require any "container" for their existence. From the formal point of view, the nonlocality of mathematics also follows from the fact that any computation is equivalent to the operation of some Turing machine, but the description of a Turing machine does not provide for any space-time binding of this device — it is a purely logical construction. In particular, the one and the same Turing machine can have different physical implementations that result from the Turing machine's work when there is no way to guess how the machine was implemented "in hardware."

Although mathematics, as we can see, possesses some interesting features characteristic of possible classical, but nonlocal substrates, this, generally speaking, does not mean that the exactly mathematics as understood in the standard way is really a substratum. In the general case, one should keep in mind the possibility of the existence of a substratum somewhat similar to mathematics, but not necessarily being mathematical in the literal sense (such an example is suggested, in particular, by the above mentioned example of the mental world). Hypothetical substrates for this type will be called pseudomathematical. Here, clarification is needed. There is a possibility that a structure, which, in the depthless analysis, appears as pseudomathematical, but actually admits a mathematical description. This case is not of interest to us, because *pseudo*-mathematical character of the structure are seeming in such a case. We mean the cases when a pseudomathematical structure is irreducible to a purely mathematical structure. Of course, the assumption for such structures is pure speculation, but such an opportunity should be borne in mind for the sake of generalizing the analysis. However, if the actual substrate is irreducibly pseudomathematical, which apparatus should be used to study it? We have no answer. Ultimately, there is no guarantee that nature is knowable up to the very end.

Despite the possibility of encountering an irreducible pseudomathematical structure, the standardly understood mathematics, in any case, remains an interesting paradigmatic example of general pseudomathematical substrates, so it makes sense to discuss, in more detail, the nature of mathematics.

About the nature of mathematics

The empirical status of the objective reality for the world of mathematical forms

Probably the most significant objection to the fact that mathematics can be an element of objective reality that plays the role of a substrate in a mass-energy world, where the mathematics exist only in the imagination of people and has, therefore, a purely mental nature. It is not an objectively existing thing, and there is only the property of human consciousness, which is its way of processing information about reality. An essential alternative to this view is provided by various forms of mathematical Platonism according to which the mathematical forms exist objectively, but they are only known by man. Between these extremes, there are many other philosophical positions and shades of positions regarding the essence of mathematics. Our position is that mathematical Platonism is not only a reasonable philosophical position, but the question about the objective existence of mathematical forms can be reduced to an experimental test, and therefore, in fact, is moved beyond the limits of philosophy and is introduced into the framework of empirical science. The point of view that will be presented below is an extremely strong mathematical Platonism, which could be called empirical mathematical Platonism (not to be confused with mathematical empiricism).

Consider some "computable mathematical form," for example, the trillionth decimal digit of the square root of 4711. It can be computed. However, the value of this decimal digit is unknown to anyone at the time of writing this article, and it is not fixed anywhere; therefore, it cannot be considered as an element of someone's consciousness, or as an element of culture in any form. However, regardless of whomever and whatever method initiates the calculation of this value, the result will be the same (if the calculation is correct[1]). This reflects the simple fact that its value existed even before someone calculated it, and it does not depend upon any subject. The only sign of the objective existence of an object is its objective cognoscibility. This is the case here: the value of the trillionth digit of the decimal decomposition for the square root of 4711 is objectively knowable, and therefore, it objectively exists.

This, however, is still only a philosophy, while the issue of the objective existence of computable mathematical forms can be given a much more rigorous empirical status. Specifically, the idea of the objective existence of the value of a computable object leads to a verifiable prediction that any correct calculation of the value of a computable object will lead to the same value. That is, it leads back exactly to the one that existed before any practical calculations were initiated. Moreover, the objective existence here turns out to be falsifiable according to Popper. Indeed, it is possible to falsify an objective existence based on the value of a computable object: it is sufficient to present two correct calculations with different results, and the objective existence will be falsified. Where Popper's criterion is applicable, the philosophy ends there and empirical science begins. An objective existence of computable mathematical forms then enters the sphere of empirical science.

To avoid misunderstandings, we note that, under computable mathematical forms, one should understand not only simple objects, such as the digits of a decimal expansions for a given square root. Computable mathematical forms are also all provable mathematical theorems. The formulation of a theorem is a logical expression that has the truth values of "true" or "false." The proof of the theorem from the formal point of view is no more than a calculation of this value. Another class of computable objects are different complex fractal sets on a plane, such as, for example, the Mandelbrot set, which was the favorite

[1] Note that checking the correctness of a computation is an algorithmically solvable problem, so the question of whether this computation is correct or incorrect can always, in principle, be given an unambiguous answer.

example of the famous mathematical platonist, Roger Penrose [Penrose, 1989, p. 92-93]. The Mandelbrot set has an incredibly complex structure at all arbitrarily small scales so that it cannot be completely computed; nevertheless, it exists quite objectively, and any fragment of it could be reproducibly studied in any detail.

Let us consider two possible objections to the above arguments.

First, everything here is reduced to mathematical proofs and calculations, but how objective is the method of mathematical proofs or calculations? After all, these were invented by people.

It can be argued that the ordinary empirical method of the cognition of nature also was invented by people, and in this sense, the method of mathematics is no worse than the empirical method for the natural sciences. The question of trust in both of these methods is a question of convention, or, phrased in another way, the question of practice, but the reality of mathematics, in this sense, is justified in a similar manner as the reality of the surrounding mass-energy world.

The second objection may be that the prediction of the uniqueness of the results for correct calculations for the same mathematical form is in fact trivial. If different correct calculations lead to different results, then, by definition, that system, where the calculations were performed, is contradictory. Therefore, if our mathematics are consistent, then such results will always be trivially the same. However, how do we know that our mathematics are consistent? This fact has not yet been proved, and there is Gödel's second theorem on incompleteness [Kleene, 1952: 204-217] to consider, which states that, if mathematics are, in fact, consistent, then this fact cannot be proved. Nature forbids us precise knowledge of the consistency of mathematics. The consistency of mathematics must be taken for granted, or, more precisely, consistency must be an empirical fact. It is appropriate to quote Nicolas Bourbaki [Bourbaki, 2004: 13]: "To sum up, we believe that mathematics is destined to survive, and that the essential parts of this majestic edifice will never collapse because of the sudden appearance of a contradiction; but we cannot pretend that this opinion rests on anything more than experience."

Predicting the unambiguity of the results of computations is equivalent to the assertion that the consistency of mathematics is true. Thus, the statement about the objective existence of computable mathematical forms is equivalent to the assertion regarding the consistency of mathematics. However, neither is trivial because of the existence of Gödel's second theorem on incompleteness.

We have, so far, confined ourselves to the objective status of the reality of computable mathematical forms. But there are mathematical forms of a different kind. These are entire mathematical systems, such as a system of natural numbers, systems of real and complex numbers, abstract Euclidean and Riemannian spaces, groups, etc. They are not simply the results of some computations, but the experience associated with computable forms suggests that these systems exist in some objective sense as well. From a purely philosophical point of view, their objective existence can be justified by the fact that their structure can be studied in any detail by an objective method, such as through calculation or with a proof. This makes them look like computable objects like the Mandelbrot set. But there is no complete analogy here, because, unlike the Mandelbrot set and similar structures, the notion of an "internal structure" of a natural number system, for example, is too vague.

It is possible to follow a different path that much more clearly emphasizes the essence of such mathematical sytems. With each system, one can unambiguously associate the proposition about the consistency of the system of axioms describing the given system. We have already seen (and it was clear, at least from the work of Hilbert and Bernays [Hilbert, 1968]) that the concept of existence and consistency in mathematics is essentially equivalent. Each proposition of consistency has a clear mathematical meaning, and is formulated as a logical expression, which must have a certain truth value. In this sense, the problem is reduced to the existence of computable mathematical forms. However, there is one essentially new circumstance. According to Gödel's second theorem on incompleteness, which is true for most of the interesting mathematical systems, if the system is consistent (and thus exists), then the truth value for the expression characterizing its consistency cannot be calculated by the means available within this system. Therefore, the consistency of such systems, and at the same time their objective existence, although it has a clear meaning, should be regarded as a purely empirical fact. Here, however, it should be noted that there are notrivial mathematical systems where the action of Gödel's second theorem does not extend. Examples include arithmetic without multiplication, as stated by Presburger [Kleene, 1952: 204], arithmetic without the principle of induction, and some others. For such systems, consistency may be proved by the simplest means of first-order logic, and thus, the objective status of the existence of such systems is computable for as simple computable mathematical forms.

Another type of object is the "nontrivial mathematical truth" that exists within the framework of Gödel's first theorem on incompleteness [Kleene, 1952: 204-217]. For any sufficiently complex mathematical system, the so-called Gödelian statements can be formulated. These statements have a clear mathematical meaning, but which can neither be proved nor disproved within the framework of this system. In practice, such true statements, in an absolutely incomprehensible way, are sometimes guessed by people using the "direct vision" method, and among such statements, there are sometimes very interesting objects. For example, in relation to standard arithmetic, the Gödelian statement is a highly nontrivial Goodstein theorem. Although it is not contained in formal arithmetic, it turned out that it can be proved if we additionally involve the so-called principle of transfinite induction (which is rather nontrivial). Until the status of the objective existence of such objects is clarified with the help of additional, specially invented methods, such as a proof (as with the case of Goodstein's theorem), it largely resembles the status of the existence of complex mathematical systems like the system of natural numbers and is purely empirical or even hypothetical.

Although the question of the objective existence of mathematical forms, in the general case, is rather confusing, in all cases, it has a solid empirical basis in the sense that their objective existence is falsifiable by representing counterexamples that show the inconsistency of objects or systems containing these objects. Thus, statements about the objective existence of mathematical forms can be either true or false, but their status is not a matter of convention or philosophy. These are questions from the sphere of empirical science.

The physical foundations of mathematics

The world of mathematical forms not only exists objectively, but, irrespective of the fact that it can be a classical nonlocal substratum of the mass-energy world, the world of mathematical forms has some simple connections with the mass-energy world. Several types of such links can be identified.

All calculations and mathematical proofs can be considered as some *process* of *information* transformation.

To organize the process, at least some material carrier for this process is needed — an object where this process, in fact, is happening. To perform calculations, such objects must be classical, which only allows one to follow the calculation in which they participate. We will call such objects calculators. The existence of classical calculators is a necessary prerequisite for the possibility of organizing processes, called calculations, without which, the concept of computation has no operational sense. The classicality of calculators is inextricably linked to the existence of a classical limit of quantum behavior. The existence of a classical limit in quantum physics is a very subtle and completely untapped property of quantum theory. It is exactly the existence of this non-trivial classical limit of the quantum world that allows mathematics to possibly exist, or, more precisely, it allows us to possibly contact our physical world and the world of mathematical forms, which are inextricably linked.

To organize the computational process, which is a linear sequence of logically related steps, the linear one-dimensional causality of our world is also important, which is related to the one-dimensionality of time and to the Lorentz causal structure. Thus, the second physical root of mathematics is the causal Lorentz structure of the world. Regardless of whether our time is multidimensional or the causal structure of space-time, the concept of linear causality could not have formed, and with it, the concept of the causal process and the notion of computation, as a particular case of such a process, would be meaningless.

The concept of information suggests that information can be presented on some material carriers for its use. Information carriers should also behave like classical systems to ensure the possibility of reading and copying information without disturbing. This, again, brings us back to the existence of the classical limit of quantum dynamics. Without classical calculators and information carriers, and without the Lorentz causal structure of space-time, mathematics could not have an operational sense in our world.

One can note a deep analogy between measuring a physical quantity and using calculations (or proofs) in mathematics. Indeed, to measure a physical quantity, you need to use a physical setup. This is a measurement device that performs the prescribed procedure to obtain the value of the physical quantity, which is the result of the measurement. To perform a calculation, you need to use a physical setup. In this case, a calculator executes the prescribed procedure and gets the value of the calculated mathematical form, which is the result of the calculation. It is easy to see that the procedures are almost identical. Formally speaking, computing is simply a kind of physical measurement. Moreover, for some calculations, the procedure reduces to performing a complex physical experiment. For example, the procedure for decomposing a large integer into prime factors, which can only be done with the help of, so far, a hypothetical quantum computer, must require a very complex physical experimental setup.

There are different definitions of what mathematics is. For example, the program by Nicolas Bourbaki is based on the notion that mathematics is a meaningful study of the *structures* on *sets* using the apparatus of *proofs* based on *mathematical logic* [Bourbaki, 2004]. The concepts that are directly related to physics are emphasized using *italics*. Structures are informational in nature and, therefore, their fixation requires the existence of a classical information carrier.

A proof relates to the existence of the classical sector of quantum theory, where causality has already been discussed above.

The notion of a set is also directly related to the existence of the classical sector of the quantum physics. If, in nature, there were no classical objects possessing qualitative certainty, then the concept of a set would simply not be formed. A counterexample is given by systems of quantum particles, such as photons, where it is often impossible to uniquely determine how many there are and the type of particle that they belong to. The concept of a qualitatively defined element of a set loses its meaning in this context. One could even say that a set is the most general physical model, namely, it is a model of the existence of qualitative certainty for objects in the classical sector of physics. Finally, mathematical logic is, in fact, classical logic as described by Aristotle, which is based on the law of exclusion of the third and on the rule of inference from *modus ponens*. However, classical logic is not *a priori* at all. It is consistent with everyday macroscopic practices associated with the classical interface between information processing systems, such as the consciousness of humans and animals, and the quantum universe. The classical sector, again, plays a key role here. In addition, the concept of logical consequence or deduction is an abstraction from the concept of linear causality, and, in fact, is the ultimate general model for the linear causality of the real world just as a *set* is a model of classical certainty.

It should be noted that the direct connection between the concepts of a *set* and logic pertaining to the existence of the classical sector of physics was understood long ago [Birkhoff, 1936]. This spawned numerous programs for quantum logic and even quantum mathematics; some of which are now used to search for models of quantum gravity [Doering, 2011; Dahlen, 2011].

Finally, it is necessary to note the mystical "unreasonable effectiveness of mathematics in the natural sciences" [Wigner, 1960], which also represents a certain connection between mathematics and physics. In fact, this "unreasonable efficiency" means that the processes of the real physical world are maps, with good accuracy, of consistent mathematical models. The reason for this is not yet clear (we will return to the discussion of this problem below), but it is obvious that some contribution to this unreasonable effectiveness stems from the foundations of mathematics, which, in fact, represents the most common physical models of classicality and causality, which was explained above. They are by no means *a priori* and they are not arbitrary.

Thus, the world of mathematical forms not only has an objective existence, but it also is not purely ideal in the sense that there are numerous connections between it and the material world and physics. The concepts of calculation, proof, set, and logic are connected to the possible existence of the classical sector of physics and with the linear causal structure of our world. The foundations of mathematics (set and logic) are not *a priori*, and it represents the most general model of the classical causal sector of physics. This suggests that mathematics is as much a real part of nature as physical (mass-energy) reality, although it exists in a different way. Existing objectively for something does not necessarily mean that the object must be placed in space-time.

The structure of quantum gravity models and mathematical substrates

Let us sum the subtotals. We have seen that reality can be divided into layers that are related to each other by a substrate-image relationship. We also saw that, in addition to the mass-energy reality, there is an objective reality of the world based upon mathematical forms, which is nontrivially related to the physics of the mass-energy world. What is the relation between mathematical and physical realities? Mathematical reality has some features that make it an interesting candidate for the role of the classical, but nonlocal substrate of the physical world, and the world of mathematics and physical reality may be related by the substrate-image relationship. More precisely, when existing objectively, not all mathematics, but some mathematical structures can be a substrate of the physical world. Then, the "fundamental ontology" of reality itself may not be "physics," but to represent, in a certain sense, an ideal information structure. The natural number set or the concept of a group in mathematics exists by virtue of logical, because these systems are consistent. The mathematical structure that provides the fundamental ontology of the real world can exist for the same reason. In this sense, our entire observable world exists because of logical necessity.

Here, it is necessary to dwell, on the one hand, on the subtle methodological distinction between commonly understood mathematics, and, on the other hand, on the mathematical structure as a substratum of the physical world. All mathematical structures for theoretical physics (and, in general, the natural sciences) are models of fragments of physical reality. Very often, a mathematical structure appears in mathematics as pure abstraction from some internal mathematical needs, but later, it is in demand as a mathematical apparatus for solving practical problems. However, the mathematical structure, as a fundamental ontology or as a substrate of the physical world, is not a model for anything other than, perhaps, itself. Actually, it distinguishes itself from mathematics in the usual sense, so much so that such an object deserves a special name. The fundamental mathematical ontology is an adequate term.

The fact that the fundamental mathematical ontology is a substratum of physical reality can be an explanation, or part of an explanation, for the "unreasonable effectiveness of mathematics in the natural sciences." There is no mystical connection between physics and mathematics, because mathematics and physics are simply one and the same thing at the level of fundamental ontology.

However, as we have already mentioned, everything can be somewhat more complicated in the sense that a fundamental ontology may not be a mathematical, but an irreducible pseudomathematical structure as defined in the section "Mathematical and pseudomathematical substrates." Then, such a fundamental structure begins to play the role of a common root for all physics and mathematics in the ordinary sense, which grew out of this root in the "low-energy limit." Physics and mathematics are simply different manifestations of one fundamental "quasi-information" entity — these are its two different sides. This may also help explain the unreasonable effectiveness of mathematics in the natural sciences, but, as we see, in a slightly different way. Mathematics and physics are united by a common origin.

If the entire physical reality is an image in a mathematical (or pseudomathematical) substrate, then this substrate is nothing else than the desired TOE, which is also quantum gravity plus a unified theory of all interactions. In this case, it turns out that quantum gravity is not quite physics. Researchers of quantum gravity usually mean that they are looking for some physical theory, and because of this, they sometimes act by analogy using already known theories. However, as you can see, quantum gravity may not be exactly where it is sought. In this sense, it is interesting to see if there is an indication of such a purely mathematical or, better yet, an informational nature behind the modern theories for quantum gravity.

Currently, there are many ways to search for the quantum theory of gravity. We will dwell on string theory, loop quantum gravity, and the theory of causal sets.

String theory claims that it is the unified quantum theory for all interactions; therefore, it describes the quantum of gravitational interaction as a graviton. Hence, in an explicit form, it is a quantum theory of gravity. From general considerations, because of the dynamic nature of general relativity, the smooth structure of space-time must be violated at scales on the order of a Planck — 10^{-33} cm, 10^{-43} s. How this violation should occur is unclear. Perhaps instead of a smooth manifold, there will be some discrete structure or "space-time foam," that will be discovered. In fact, in different theories, both of these possibilities and their various combinations are realized, but it is quite clear that there cannot be a smooth manifold. Nevertheless, in string theory, it is assumed, from the very beginning, that objects on the order of the Planck scale — strings or superstrings — move in a smooth space-time background. String theory, in fact, studies the quantum dynamics of strings in this context. Then, it turns out that, for the self-consistency of the theory, it is necessary for this space-time background to be ten dimensional. From this, a widely-circulated statement occurs, where string theory leads to the conclusion that our space-time has more than four dimensions.

However, it is impossible to speak about any smooth space-time manifold on the scale of strings. What, then, is this smooth multidimensional manifold, where the dynamics of strings could be studied? In our opinion, the answer is simple: this manifold has nothing to do with our physical space. This is a purely formal background, where, using pure formality, something that looks like the dynamics of one-dimensional objects, such as strings, is studied. Our physical space, together with all the real dynamics of the fields and particles within it, should turn out to be an interpretation of these formal dynamics. Real physics must have an emergent character with respect to purely formal string dynamics, so, in this case, the substrate-image relationship is explicitly realized. String theory could be a mathematical substrate of the physical world if string theory is true.

Judging by the current state of string theory, it is still very far from understanding the detailed structure of this map. String theory is still developing primarily as a perturbative theory, and by analogy to conventional quantum field theory. In this way, it is probably necessary to sum all the orders of perturbation theory to obtain a coherent state of gravitons, which (maybe) can be interpreted as a real physical space. However, even the finiteness of string theory, with all its orders of perturbation theory, has not yet been proven, so it is difficult to even pose a question about summing the series.

Loop quantum gravity is a more-or-less consistent attempt to find the canonical form for the quantization of general relativity. To perform this quantization, the theory of gravitation is reformulated as the so-called loop variables, hence the name of the theory. Loop gravity, from the very beginning, was a nonperturbative theory, and it is also a background-independent theory (in contrast to string theory, which is devoid of both qualities). The result of quantization is that a structure is purely combinatorial in nature. This is the so-called abstract edge-labeled graph. Quantized elements in the volume of space are placed at its vertices, where the edges represent the quantized elements of surfaces that delimit different volume elements. Although, to each vertex of the graph, there corresponds a certain amount of volume, and, to each edge - the area - these quantities are no more than numbers, and this abstract graph should not be considered embedded to a metric space. It, we shall repeat, is a purely discrete combinatorial structure. The edges of the graph correspond to numbers that run through the possible values of the spin, e.g., 0, 1/2, 1, ... (the edges are labeled with these numbers); therefore, such graphs are also called spin networks. The value of the spin attributed to the edge determines the quantum of area corresponding to the same edge. The central role of the theory is played not so much by the spin networks themselves, but by the amplitudes of the transition between different networks, which is coded the quantum dynamics of space-time. There are different ways of transforming loop quantum gravity, which, in its original formulation, is exclusively a theory of quantum geometry in space-time, into a unified field theory. This is achieved, for example, by replacing graphs with so-called ribbon graphs by preserving a significant part of the rest of the formalism [Bilson-Thompson et al., 2007]. Ribbon graphs replace the edges of the graph with ribbons, which can be twisted in different ways, where an additional topological degree of freedom arises in such a system. The twistiness of the ribbons encodes different types of particles that move in space-time. Such objects are still not metric because the topology of the twists is determined in a purely combinatorial way.

Thus, loop quantum gravity assumes that the fundamental ontology of physics is something that has an almost purely abstract informational nature. It does not completely have this nature because the theory retains the dynamics represented by the transition amplitudes between different spin networks, which are like the usual amplitudes of quantum mechanical transitions. We are at a loss at how to say how much such a dynamic violates the “abstractness” of the system, but some features of the abstract mathematical substrate are obvious. All the observed physics and space-time appear, in theory, in an emergent fashion, which is as it should be with respect to the substrate-image relation.

A very similar situation takes place in the theory of causal sets, although the ideas embedded in this theory are different. The theory of causal sets is not a consistent quantization of general relativity, and from the very beginning, it assumed that space-time has a discrete and, at the same time, a purely combinatorial nature based on causality. Specifically, it was assumed that, at the fundamental level, space-time is a set of structureless objects (events) that are related only to the causality relation: for each set of two events, either the first causally precedes the second, or the second precedes the first, or the causal relationship between them is absent. The causality relation establishes, for a set of events, a partial order relation from which it is possible to obtain a surprisingly rich set of consequences. The theory of causality sets is currently one of very few theories (perhaps the only one) that naturally predicts the small value of the cosmological constant [Ahmed, 2004]. The metric space, as the foundation of the theory, is absent, but it should appear emergently in the low-energy limit just as it did for loop gravity. For the causal sets, quantum dynamics is determined in various ways; for example, in terms of the Feynman sum of histories. Regarding the connection between the theory of causal sets and the notion of information or the mathematical substrate for the physical world, it is possible to repeat exactly the same thing that was said for loop quantum gravity. The structure of the theory is very close to the definition of a mathematical substrate.

In quantum gravity, several more models can be found that give examples of the same property: theories for different types of triangulation, Regge calculus, and some others. All this indicates is that the vast majority of quantum gravity theories demonstrate a general tendency: the underlying structures become so abstract that they begin to resemble purely mathematical combinatorial or informational constructs, which makes them related to the concept of a mathematical substrate or to the concept of a fundamental mathematical ontology. Although such terms are usually not introduced explicitly, but in fact, their movement is rather close to this direction.

“It from bit” was said by John Archibald Wheeler [Wheeler, 1990], but this article only heuristically substantiated one of the ways that one could come to this idea. Thought is not new and it is present in modern literature through a variety of options. However, the ways that different authors lead to this idea are different.

Wheeler came to this concept mainly through the concept of physical measurement — mainly quantum measurements. He was looking for the answer to the question: what is reality? Everything that is considered a reality by us is a stream of information that reaches our consciousness, and the information flow is decomposed into elementary acts of “observer-participancy,” which are characterized by exactly one bit of information 0/1. Therefore, the basis of reality is informational, and not just informational, but discrete informational. Continuous space-time is an illusion — this should be followed by a discrete informational-theoretical structure. String theory is an interesting physical theory, but it must also be reformulated in discrete terms. The concept of the observer (or his consciousness) is also central to this concept, without which, the very idea of the elementary act of participation by an observer becomes meaningless. The universe acquires reality through an act of self-observation through the consciousness of the observer, and through elementary acts of introspection.

The way to "it from bit" by Max Tegmark is very interesting [Tegmark, 2004]. He considers the general concept of the multiverse and distinguishes 4 different levels of this category: multiverses levels I, II, III and IV correspond to their increasing "degree of hypotheticality." The first three levels are, respectively: I — the causally independent areas of our "local cosmological bubble;" II — different universe-bubbles (possibly with different physics) in the sense of cosmological models for chaotic inflation; III — parallel classical branches of the quantum universe from the perspective of the many-world interpretation by Hugh Everett. A multiverse of level IV is a mathematical multiverse, which is the most interesting in the context of this article.

The basic premise of Tegmark is the assumption that an ontological foundation of the physical world can really be a mathematical structure in the sense that the real physical world is simply isomorphic to this mathematical structure. In doing so, he emphasizes the difference between the mathematical structure, which in some ways represents all aspects of the world, and the usual mathematical models of physics, which describe only some approximately limited situations. This exactly corresponds to the difference between the fundamental mathematical ontology and the mathematical models, which also appeared above in our discussions. However, if our physical reality is one of the mathematical structures (it is isomorphic to it), then this entails a "fundamental, unexplained ontological asymmetry built into the very heart of reality, which splits mathematical structures into two classes: those with and those without physical existence" [Tegmark, 2004]. To overcome this problem, Tegmark introduces the notion of "mathematical democracy," where all non-contradictory mathematical structures are, in fact, physical worlds. Thus, a level IV multiverse is simply a collection of all possible consistent mathematical structures. Here, it is not so much "it from bit" as it is "it is bit."

The concept of a level IV multiverse sheds light on two questions. Firstly, this is the question of the "unreasonable effectiveness of mathematics." Mathematics is so unreasonably effective in describing the physical world because the physical world is some mathematical structure up to isomorphism. Secondly, why is the universe exactly the way that we see it (including the problem of fine-tuning the constants)? The language for a level IV multiverse is reformulated as a question of why this particular mathematical structure and not the other. The answer to this question is easily given in the spirit of a weak anthropic principle: our mathematical structure is simply one of those that are compatible with the existence of a thinking observer in it. Considerations from measure theory suggest that this is most likely one of the simplest structures of this type.

It is not difficult to see that, within the concept of the pseudomathematical fundamental ontology used in this article, there is much in common with the level IV multiverse described by Max Tegmark. In both systems, there is a certain way of solving the problem of the unreasonable effectiveness of mathematics, but there are differences. In Tegmark, the physical universe *is* a mathematical structure up to an isomorphism. We used the notion of a substrate-image relationship, which, generally speaking, does not have to be reduced to an isomorphism, and generally, it is not reduced to the notion of mapping. The emergence that has a place here can be substantially more complex. For example, it may resemble the appearance of the classical sector in quantum theory. Although the classical sector appears emergently on the top of the quantum fundamental ontology, but the way it emerges is not simply reduced to some mathematical map of quantum reality to classical things. In addition, a fundamental ontology may not be a mathematical structure, but something irreducibly pseudo-mathematical. In this sense, our constructions have a somewhat higher generality than Tegmark's mathematical multiverse. On the other hand, following the way of Tegmark, we can introduce the concept of pseudomathematical democracy. Then, Tegmark's mathematical multiverse is included in the pseudomathematical multiverse as its subset.

Summary

The idea that, behind the scene of the physical world, there may be some ideal information structure is very popular, although different authors came to it in different ways. Our way went through the idea of the computability of quantum theory and, then, through criticism of locally classical substrates, which led to the notion of layers of reality and that nonlocal-classical substrates turned out to be very close to mere abstract mathematical structures. Whatever the source of this idea, the results indicate that the fundamental ontology of quantum gravity may not be completely physics. Awareness of this circumstance can affect the search for the Final Theory. A simplified scheme of the relations between the notions introduced in this paper is shown in figure 1.

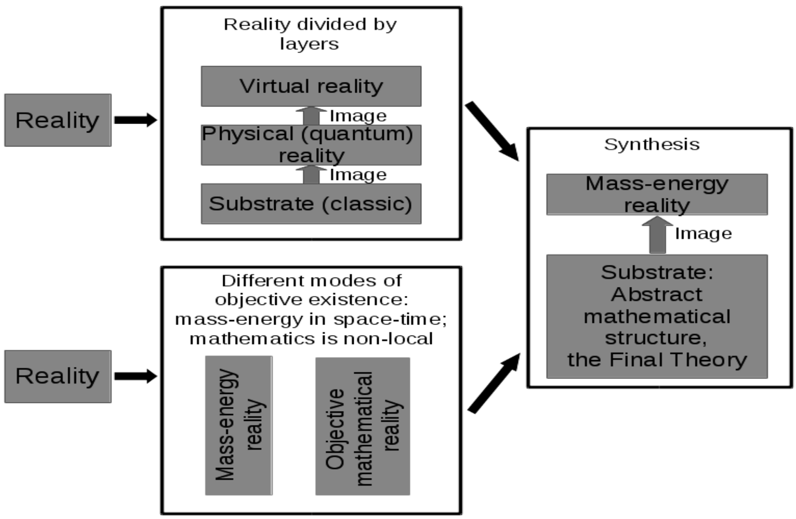

Figure 1. A simplified diagram of the relations between the notions introduced by this paper. Main simplification is that the Final Theory is considered to be mathematical, but not pseudomathematical. In the last case the usual mathematics also should be a result of an image on the pseudomathematical substrate and the mass-energy reality together with the mathematics have another meaning as "low-energy" limits of the fundamental pseudomathematical structure.

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