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Chaos in different far-off cosmic rays: a fractal wave model

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Abstract

The air shower arrival time intervals (ASATIs) were observed for several years at five different far-off stations to study the chaotic feature of the cosmic rays. The total number of chaotic time series of ASATIs is 99 from February 1994 to November 2000. The chaotic ASATIs appear six times simultaneously between two different stations, showing a higher probability of simultaneous chaos detection than the random occurrence as implied by the fractal wave pattern of cosmic ray clouds. The average fractal dimension of the fractional chaotic ASATIs for each station is 3.7 ± 0.1 pm with average duration of 23 h. The distribution of the representative (central) right ascension of air showers among 99 chaotic ASATIs has two peaks around 4 h and 20 h. The time variation of the chaotic feature of the ASATIs for several days shows a quasi-periodic behaviour with the rotation of the earth. These results seem to indicate that the chaotic cosmic rays arrive at the earth, forming wave trains like a fractal pattern which covers an area of the galactic plane surrounding the earth.

1. Introduction

Continuous observations of cosmic ray air showers with average primary energy of 3×10^{14} eV have been made in several different stations in Japan since 1996. The time sequence of air shower arrival time intervals (ASATIs) containing several hundreds of events was analysed to find the low correlation dimension as the chaos signature by use of the Grassberger–Procaccia method [1] after a fundamental check of the non-periodicity by Fourier analysis. Chaotic features were found unambiguously in several groups of ASATIs [2–4].

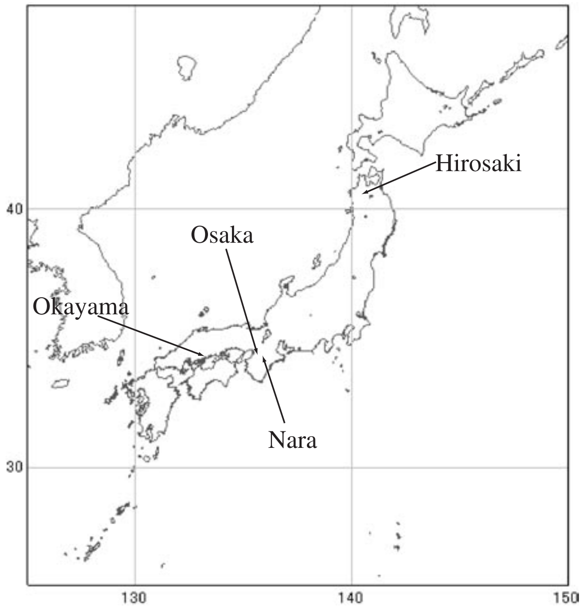

Figure 1. The locations of the five stations: Kinki University is in Osaka, Nara Sangyo University is in Nara, Okayama University and Okayama University of Science are in Okayama and Hirosaki University is in Hirosaki.

Long-range ASATIs are normally considered to be random, and actually that is true for most of them. However, chaotic ASATIs are found sometimes over one or two days, and some of them are found of shorter duration less than 12 h apart. Even in these 10 h or so, the right ascension of the observed direction changes much more according to the rotation of the earth, indicating a spatial pattern of the chaotic ASATIs. The question why cosmic ray chaos exists at all must be answered on the basis of a model which explains the above characteristics of the chaotic ASATIs. In the present paper, we reconfirm the chaotic feature of the cosmic rays which appears in some groups of ASATIs observed at different stations to confirm the simultaneous detection of the chaotic ASATIs and we add some more information on the index of variation of fractal dimension (to be defined later in section 2) obtained from every station, and a preliminary model is proposed to match the observational requirements. The simultaneous detection of the chaotic ASATIs in different far-off stations and the quasi-periodicity in the index of variation of dimension seem to suggest a cosmic ray density wave pattern of self-similar fractal structure coming onto the earth from certain directions.

2. Observations and analysis

The ASATIs used in the present study as the primary data for detecting the chaotic features have been observed simultaneously and continuously at five far-off stations. The ASATIs have been observed at Kinki University since January 1994, at Nara Sangyo University since July 1996, at Okayama University and Okayama University of Science since September 1996 and at Hirosaki University since November 1998 [5]. From Kinki University (Osaka), the Nara Sangyo University is located about 11 km south-east, Okayama University and Okayama University of Science are located about 153 km and about 152 km west, respectively, and Hirosaki University is located about 787 km north-east as shown in figure 1.

The average event rates (events $\min^{-1}$) at each station are 0.320 at Kinki University, 0.295 at Nara Sangyo University, 0.411 at Okayama University, 0.530 at Okayama University of Science and 0.412 at Hirosaki University, depending upon the effective aperture of the installation. The total number of observed ASATIs was about 3×10^6 events upto December 2000 at the five stations. The air shower array in each station consists of 5–8 plastic scintillation counters ($50 \times 50 \times 5$ cm^3). The arrival times are recorded with a time resolution of 10 ms using the GPS system. The arrival direction of the air shower is calculated with an arrival time difference between each counter obtained by a TDC (time to digital converter), on assumption of a plane air shower front. The distribution of the zenith angle θ in all the datasets can be fitted to a function $(\cos\theta)^n$, with $n = 9.7 \pm 0.1$. The frequency of air showers triggered in 10 min intervals obeys the Poisson distribution well, though air shower clusters were mostly observed around 5 h and 20 h of right ascension [6]. However, the air shower clustering is not concerned with the chaotic feature of the ASATIs.

2.1. Fractal dimension analysis

The ASATIs, whose number is *N*, can be designated as $\phi_i(t_j)$, where t_j denotes an address time to which the *i*th ASATI ϕ_i $(i = 1, 2, \ldots, N)$ belongs. In this notation, the time of the *n*th air shower detection after t_j is given by $t_j + \sum_{k=1}^{n-1} \phi_k(t_j)$. Thus, we have an *m*-dimensional embedding space in which the *m*-dimensional vector points,

$$\vec{\phi}^{(m)}(t_j) = [\phi_1, \phi_2, \ldots, \phi_m](t_j) \qquad (j = 1, 2, \ldots, N - m), \tag{1}$$

are embedded. The distribution of the vector points in the embedding phase space has a self-similar (fractal) structure and represents the characteristic properties (the fractal correlation dimension, in particular) of the dynamical system, if the original ASATIs had a chaotic feature.

Figure 2(a) shows the result of the analysis according to the Grassberger–Procaccia method for a set of 300 typical chaotic ASATIs embedded in 11-dimensional phase space, showing the fractal dimension D_m of the system to be 3.9. The sampling number $N = 300$ is chosen rather empirically for the reason that the chaotic ASATIs of relatively short duration of about 15 h contain about 300 ASATIs. If the Eckmann–Ruelle criterion ($D_{\max} = 2\log N$) [7] is applicable to the cosmic ray chaos, the maximum detectable correlation dimension $D_{\max}$ would be about 5. Using the embedding dimension $m = 11$ in accordance with the Takens criterion ($m > 2D_{\max} + 1$) [8], we have detected the correlation dimension up to 4.8 as shown in table 1. The symbol $C_m(r)$, the correlation integral, denotes the total number of pairs of vector points separated by a distance less than *r* in the *m*-dimensional embedding space. The fractal dimension D_m is given by $\mathrm{d}\ln C_m(r)/\mathrm{d}\ln r$ and smoothing out local irregularities with a kind of maximum entropy method. The fluctuation of the D_m-value is less than ± 0.1 in the range of the constancy. So, the constancy of the D_m-value in some finite range of *r* indicates that $C_m \propto r^{\text{constant}}$ in the range of *r*. The 300 chaotic ASATIs are detected so that the constancy of the D_m-value is less than 5 and the range of the constancy is larger than 1/5 of the total range of $\ln r$ to exclude the possibility of accidental appearances of self-similar structures. In the case of random ASATIs, the constancy of the D_m-value does not appear over a wide range (see figure 2(b)). The disappearance of such constant D_m ranges with surrogate data by random disordering of the same set of ASATIs also distinguishes the chaos from coloured noise. The surrogate data test [9] is applied 100 times to each chaotic ASATI which has the constancy of D_m-value. The appearance of a similar constancy of D_m-value is twice at most for a 100 tests, so they are considered to be chaotic rather than coloured noise. Aglietta *et al* [10] reported that the air shower data with different sampling, corresponding to about one year of measurement, have a feature of coloured noise.

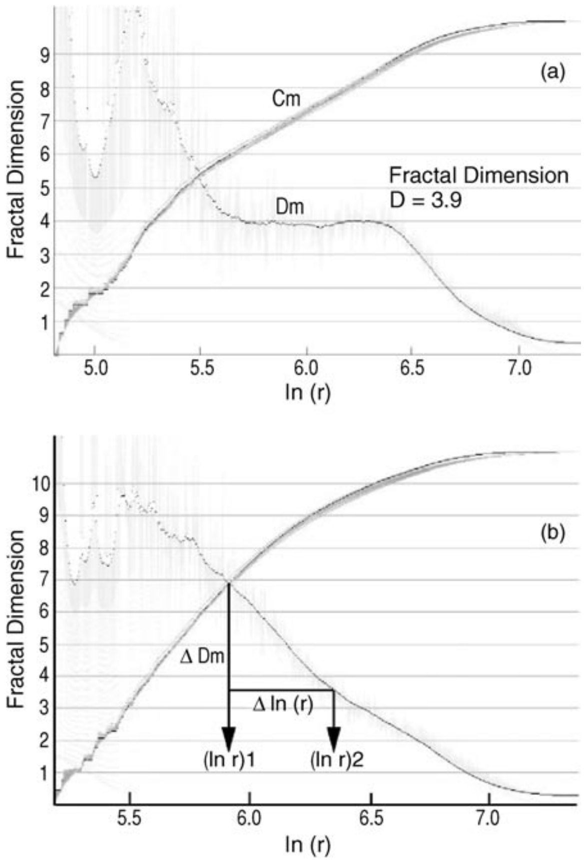

Figure 2. Examples of the diagram of the fractal dimension analysis for (a) the 300 chaotic ASATIs and that for (b) the 300 random ASATIs. C_m is the correlation integral (number of point-pairs of the mutual distance less than r in embedding space), and D_m is the correlation dimension (power in r). The flat level of the D_m-value in the chaotic case (a) indicates the fractal dimension.

The analysis is executed for every 300 ASATIs on the series of ASATIs among 3×10^6 events, shifting the first event by 10 events ($\sim$30 min). Then, the precision in the determination of the centroid of the period of the chaotic ASATIs is ± 0.5 h. The chaotic ASATIs are considered to have fractal correlation among the whole series of ASATIs. So, even if the chaotic feature is relieved step by step by the shift, mixing of random ASATIs, it does not mean that only the shifted 10 events are chaotic. Tables 1–3 give the 99 detected cases of chaotic series of ASATIs which have fractal correlation among the ASATIs observed at the five stations since January 1994. The second column in these tables shows the date and time of the first event of chaotic ASATIs, the third column shows the fractal dimension, the fourth column shows the central right ascension, the fifth column shows the duration (hours) of each chaotic case, the sixth column shows the duration of the simultaneous detection of the chaotic ASATIs, shown in the fifth column, between two stations and the name of the other station. Generally, the value of the fractal dimension is raised and comes to have m dependence by a partial mixing of noise, as confirmed by the simulation by use of the artificial discrete chaotic data. Therefore, after detecting the chaotic long series of ASATIs which include more than 300 events, the fractal dimension and the representative (central) right ascension can be estimated

Table 1. The chaotic ASATIs detected at Kinki University.

Year	Date D M H	Fractal dimension	Right ascension (h)	Duration of chaos (h)	Simultaneous chaos (h)
1994	23 Feb 01	4.5	8.4	20	
	30 Apr 08	4.3	16.2	23	
	23 Jun 13	3.0	7.6	55	
	4 Dec 21	3.8	20.3	25	
1995	5 Feb 09	4.2	16.9	34	
	20 Mar 03	4.6	17.1	31	
	19 Oct 11	4.2	13.7	36	
1996	16 Jan 12	3.5	18.5	26	
	29 Jan 05	3.4	13.7	29	
	29 Apr 12	3.1	2.6	35	
	5 Jul 18	2.9	7.9	24	
	25 Jul 18	4.6	3.7	23	22 (Nara)
	12 Nov 10	3.3	20.5	32	
	4 Dec 03	3.0	9.9	25	11 (Okayama)
1997	3 Feb 16	4.8	19.6	16	
	15 Apr 02	3.5	5.0	20	
	8 Jun 01	4.0	15.0	18	
	5 Sep 03	3.6	21.8	25	
	7 Oct 17	3.7	2.6	31	
	10 Nov 01	2.8	1.8	26	
	9 Dec 06	4.3	11.3	25	
1998	6 Feb 00	3.2	22.1	20	
	7 Mar 22	3.7	22.5	16	
	17 Jun 09	2.9	17.6	18	
	21 Aug 21	3.8	15.1	23	14 (OkaySci)
	17 Sep 15	4.4	0.6	20	
	28 Oct 01	3.8	23.7	26	
	27 Nov 06	4.0	4.8	20	
	2 Dec 08	3.4	14.2	33	
	8 Dec 08	4.7	13.3	26	
1999	12 Feb 17	3.4	21.6	17	
	29 Mar 04	3.9	7.1	33	
	1 May 14	3.3	21.1	17	
	21 Sep 07	3.8	20.7	19	
	13 Oct 11	3.2	11.5	31	−33 (Okayama, OkaySci)
2000	8 Jan 00	3.6	4.6	43	
	17 Jun 03	3.7	21.4	59	
	Average	3.7		27	

by use of the best (dominant) 150 or 200 ASATIs which have the smallest saturation point of D_m-value at $m = 11$ among the original long ASATIs with increasing embedding dimension m that arises from the effective dilution of the random component.

On the other hand, ten cases of the 300 chaotic ASATIs are detected in 3×10^6 random ASATIs that are artificially generated by the computer, in comparison to 99 cases of the observed 3×10^6 ASATIs. This result means that the chance probability of the appearance of the 300 chaotic ASATIs is about 1/10th of the practical appearance of the chaotic ASATIs, though the computer generated random ASATIs do not necessarily guarantee pure randomness for the number of short-range data as short as 300. Moreover, three cases in tables 1 and 2,

Table 2. The chaotic ASATIs detected at Nara Sangyo University and Okayama University.

Year	Date D M H	Fractal dimension	Right ascension (h)	Duration of chaos (h)	Simultaneous chaos (h)
			Nara Sangyo University		
1996	25 Jul 16	3.7	5.9	24	22 (Kinki)
	30 Jul 11	3.7	15.6	24	
1997	17 Jul 10	3.6	7.8	30	
	19 Jul 16	4.8	8.2	25	2 (OkaySci)
	5 Aug 17	4.2	4.3	20	
	13 Nov 23	3.6	0.5	27	14 (Okayama)
	23 Nov 16	4.0	19.4	32	
	19 Dec 22	3.0	6.4	25	
1998	3 Apr 20	4.1	2.9	34	
	4 May 03	3.5	12.6	32	
	12 Dec 05	3.4	24.0	17	
1999	28 Jul 07	3.9	20.5	22	
2000	18 Jan 04	3.7	7.7	30	16 (Okayama)
	15 Jun 02	3.5	21.1	31	
	28 Aug 15	4.1	4.6	22	
	28 Oct 17	2.2	18.8	36	
	Average	3.7		27	
			Okayama University		
1996	18 Nov 22	4.1	19.6	17	
	3 Dec 21	4.1	22.3	17	11 (Kinki)
	12 Dec 21	3.3	22.5	23	
	18 Dec 06	4.1	6.0	18	
1997	27 Apr 03	3.4	18.1	47	
	10 Jun 02	4.2	16.5	18	
	23 Jul 11	3.0	19.3	15	
	14 Nov 12	3.2	8.9	21	14 (Nara)
	28 Dec 13	3.1	16.9	24	
1998	14 May 08	4.2	17.1	13	
	6 Jul 04	3.4	18.1	17	
	20 Aug 09	2.5	21.5	18	−27 (Kinki, OkaySci)
	21 Dec 23	4.1	0.2	18	
1999	17 Apr 18	3.6	0.9	13	
	17 Aug 09	3.3	1.5	24	
	11 Oct 11	4.1	6.2	15	0 (OkaySci)
2000	18 Jan 18	4.1	18.1	21	16 (Nara)
	16 Nov 18	3.9	18.9	15	
	Average	3.6		20	

the chaotic ASATIs on 23 June 1994 and 17 June 2000 at Kinki University and on 27 April 1997 at Okayama University, have a duration more than three times the unit duration of the 300 chaotic ASATIs and also 22% of 99 chaotic ASATIs have more than twice the duration of the unit duration of 300 ASATIs. These chaotic cases with long duration contain the sequence of several chaotic ASATIs. The unit durations are about 17 h, 16 h, 12 h, 10 h and 12 h for Kinki University, Nara Sangyo University, Okayama University, Okayama University of Science and Hirosaki University, respectively. The chaotic 2×300 ASATIs

Table 3. The chaotic ASATIs detected at Okayama University of Science and Hirosaki University.

Year	Date D M H	Fractal dimension	Right ascension (h)	Duration of chaos (h)	Simultaneous chaos (h)
		Okayama University of Science			
1996	26 Oct 18	4.2	12.8	11	
	26 Nov 05	4.1	20.8	10	
	29 Dec 06	4.1	4.7	12	
1997	15 Jan 19	2.7	18.1	14	
	18 Feb 19	3.2	19.6	12	
	22 Mar 03	3.7	10.7	15	
	22 Apr 08	3.7	14.7	14	
	19 Jul 08	4.1	17.4	10	2 (Nara)
1998	2 Jan 22	4.3	14.4	28	
	6 Aug 05	4.3	0.5	27	
	22 Aug 06	3.9	19.8	27	14 (Kinki)
	18 Nov 19	4.2	3.1	25	
1999	10 Jan 16	2.6	13.3	12	
	12 Oct 02	3.5	15.9	17	0 (Okayama)
	2 Nov 03	4.3	19.3	17	
	17 Nov 08	4.1	2.9	11	
	20 Nov 19	3.8	19.0	20	
	20 Dec 11	4.2	9.6	16	
2000	01 Mar 08	4.2	7.0	10	
	Average	3.8		16	
		Hirosaki University			
1998	24 Nov 01	3.9	19.7	13	
	4 Dec 00	3.8	0.3	14	
	16 Dec 05	4.2	3.9	13	
1999	12 Jan 17	3.8	21.8	18	
	18 Mar 16	4.4	2.2	16	
	1 Jul 13	3.9	2.5	16	
	19 Sep 19	2.7	10.3	13	
2000	28 Jun 12	3.9	5.4	20	
	02 Nov 16	3.5	18.1	25	
	Average	3.8		16	

may happen by chance with a probability less than $(1/10)^2$th of that of the observed chaotic ASATIs.

The average fractal dimension for each station is consistent (3.6–3.8). It seems to indicate that the chaotic feature comes from the same nonlinear acceleration dynamics which has a degree of freedom around 3.7. The average right ascension of 11 air showers around the centre of the dominant 150 or 200 ASATIs, whose duration is about 8 h, is assumed to be the representative right ascension of the chaotic air showers. Figure 3 shows the frequency distribution of the representative right ascension of the air showers belonging to 99 chaotic ASATIs. By the circular statistics [6, 11], the chance probability of the appearance of the two broad peaks is calculated to be 1.8% which is small enough to indicate the significance of the two peaks. Such peaks should not appear if the chaotic feature comes from the probability of chance or stochastic processes. The two peaks seem to lie around the right ascension 4 h

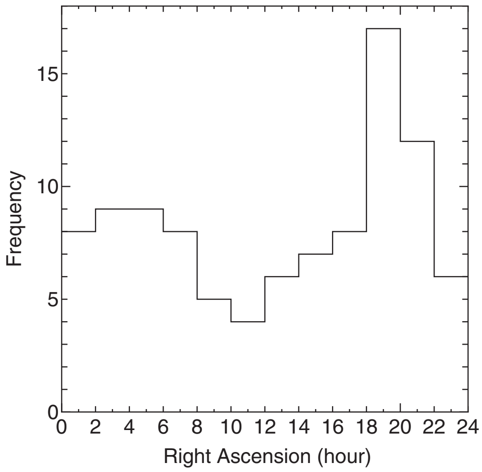

Figure 3. The frequency of the representative (central) right ascension of the dominant ASATIs among the 99 cases of chaotic ASATIs. The distribution seems to have two peaks around the right ascension 4 h and 20 h.

and 20 h. There is no correlation between the distribution of the detected fractal dimensions and the observed right ascensions. These two directions of the arrival of cosmic rays are the directions where the galactic latitude is zero, consistent with our result reported before [2], and also with our result of the cluster analysis for detecting clustering of cosmic ray directions [6], though the clustering does not coincide with the appearance of the chaotic ASATIs. These results again indicate that the chaotic cosmic rays come roughly from the direction of the galactic plane.

The chaotic ASATIs appear simultaneously six times between two different stations as shown in the sixth column of tables 1–3. One of them was detected on around 21 August 1998 at Kinki University and Okayama University of Science, the duration of the simultaneous detection of the chaos is 14 h between each duration of the chaotic ASATIs, 23 h and 27 h, the fractal dimensions are 3.8 and 3.9, the central right ascensions are 15.1 h and 19.8 h respectively.

The chance frequency of the simultaneous appearance of the chaotic ASATIs between two stations, *index*1 and *index*2, is estimated with the formula

$$\frac{n_1}{T_c} \times \frac{n_2}{T_c} \times \left(\frac{1}{10}\right)^{\frac{\sum a_1}{\tau_1 \times n_1} - 1.0} \times \left(\frac{1}{10}\right)^{\frac{\sum a_2}{\tau_2 \times n_2} - 1.0} \times (\tau_1 + \tau_2 - \tau_s) \times T_c \tag{2}$$

where T_c is the total common duration of the observed ASATIs of each station, n_1 and n_2 are the numbers of chaotic ASATIs detected during T_c at each station, τ_1 and τ_2 are the unit durations of the 300 chaotic ASATIs at each station, τ_s is the duration of the simultaneous detection between τ_1 and τ_2 at the simultaneous appearance of the chaotic ASATIs, $\sum a_1$ and $\sum a_2$ are the total durations of the chaotic ASATIs detected during T_c at each station. The n_1/T_c and n_2/T_c mean the appearance probability of the chaotic ASATIs to the hour on the assumption that the chaotic ASATIs can appear uniformly in T_c. The terms $\left(\frac{1}{10}\right)^{\frac{\sum a_1}{\tau_1 \times n_1} - 1.0}$ and $\left(\frac{1}{10}\right)^{\frac{\sum a_2}{\tau_2 \times n_2} - 1.0}$ concern the chance frequency of the appearance of the chaotic ASATIs whose duration is longer than the unit duration, 300 ASATIs, taking into account that the chance probability of the appearance of the 300 chaotic ASATIs is less than 1/10th of the practical appearance as mentioned above. If the duration of every detected chaotic ASATI is equal to

Table 4. The estimated chance frequency of the simultaneous detection of the chaotic ASATIs.

Sta.1-Stat.2	Date	T_c	n_1	n_2	τ_1	τ_2	τ_s	$\frac{\sum a_1}{\tau_1 \times n_1}$	$\frac{\sum a_2}{\tau_2 \times n_2}$	Chance det. frequency
Kinki–Nara	25 Jul 1996	23 951 h	15	13	16	17	16	1.77	1.46	0.010
Kinki–Okayama	4 Dec 1996	33 441 h	23	18	16	12	11	1.63	1.61	0.012
Kinki–OkaySci.	21 Aug 1998	32 222 h	20	16	16	10	10	1.54	1.53	0.014
Nara–Okayama	13 Nov 1997	20 211 h	13	13	17	12	12	1.63	1.47	0.011
Nara–Okayama	18 Jan 2000	20 211 h	13	13	17	12	12	1.63	1.47	0.011
Nara–OkaySci.	19 Jul 1997	20 050 h	12	12	17	10	2	1.59	1.61	0.011

the unit duration (300 ASATIs), then $\frac{\sum a_i}{\tau \times n_i}$ is calculated to be 1. Therefore, the terms are the correction factors of n_1/T_c and n_2/T_c for the appearance probability of the chaotic ASATIs whose duration is longer than 300 ASATIs.

Table 4 shows the statistical data for the six cases of simultaneous detection of chaotic ASATIs. The first column shows the two stations where the simultaneous chaos is detected, the second column shows the date of the simultaneous detection. The last column shows the chance frequency of the simultaneous detection of the chaotic ASATIs estimated by formula (2) using the data shown in the columns (the third column–the tenth column) which are mentioned above. The estimated values of the chance frequencies are small enough to indicate the possible significance of the simultaneous detection of the chaotic ASATIs. The result of Kinki–Okayama for instance means that the simultaneous detection could happen by chance only once in 318 years. On around 21 August 1998, three chaotic ASATIs were detected during less than 27 h at three different stations, Kinki–Okayama–OkayamaSci. A chaotic ASATI was detected simultaneously between two of them (Kinki–OkayamaSci.) during 14 h. In this three-fold case, the chance frequency is estimated with the formula

$$\frac{n_1}{T_c} \times \frac{n_2}{T_c} \times \frac{n_3}{T_c} \times \left(\frac{1}{10}\right)^{\alpha_1} \times \left(\frac{1}{10}\right)^{\alpha_2} \times \left(\frac{1}{10}\right)^{\alpha_3} \times (\tau_1 + \tau_2 + \tau_3 - \tau_s)^2 \times T_c \qquad (3)$$

where α_1, α_2 and α_3 are the correction factors for $n_1/T_c, n_2/T_c$ and n_3/T_c respectively, $\alpha_1 = \frac{\sum a_1}{\tau_1 \times n_1} - 1.0$, $\alpha_2 = \frac{\sum a_2}{\tau_2 \times n_2} - 1.0$, $\alpha_3 = \frac{\sum a_3}{\tau_3 \times n_3} - 1.0$, τ_s is the average of the duration of the simultaneous detection and the interval of the close detection of the chaotic ASATIs between two stations, 14 h, −18 h and −27 h. The minus sign means that two chaotic ASATIs have a close interval. T_c is 30 921 h in this case. The result of the estimated chance frequency is 0.0003 which means that a three-fold close detection like this case could happen by chance once in 10^4 years. A similar three-fold ASATI is closely detected during less than 33 h on around 12 October 1999 at the same stations.

2.2. The time variation of the chaotic features of the ASATIs

Both the mixing of chaotic ASATIs with random ASATIs and the gradual variation of fractal dimension during a chaotic period may occur in general. In order to analyse such a mixed state, we introduce an index representing the relative variation of the degree of the stochastic mixing. The index is tentatively taken to be the *slope* of the D_m-curve, *slope* $\equiv \Delta D_m/\Delta \ln r$, where ΔD_m is the change of D_m-values from $\ln r = (\ln r)_1$ to $\ln r = (\ln r)_2$, as shown in figure 2(b). In this case $(\ln r)_1$ is larger than the minimum $(\ln r)_{\min}$ by 1/3 of the total range $[(\ln r)_{\max} - (\ln r)_{\min}]$, and $(\ln r)_2$ is larger than $(\ln r)_1$ by 1/5 of the total range. According to this definition, the *slope* is zero for mono-dimensional chaos (see figure 2(a)) and is more than 5 for stochastic noise (see figure 2(b)). It generally increases with the mixing of noise, but

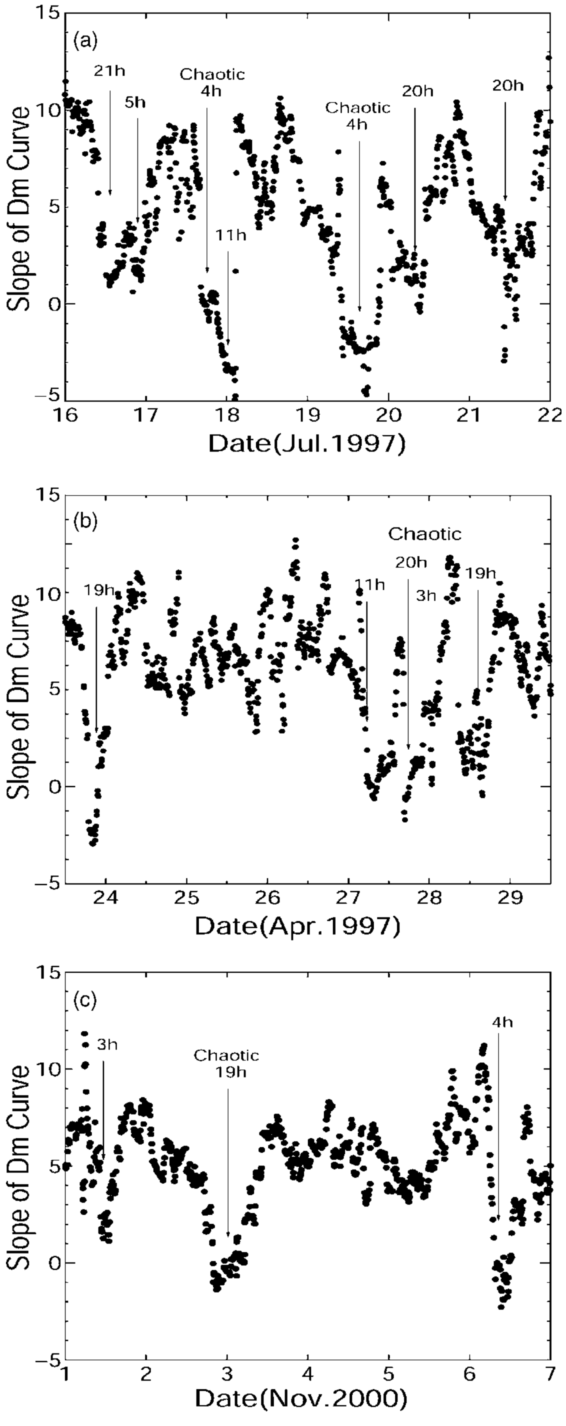

Figure 4. The typical time variation of the *slope* of the D_m-curve of each 150 or 200 ASATIs, estimated for ASATIs covering the chaotic ASATIs: (a) observed at Nara Sangyo University in July 1997, (b) observed at Okayama University in April 1997 and (c) observed at Hirosaki University in November 2000. The time variation of the *slope* frequently shows quasi-periodic troughs around the right ascension 4 h and 20 h.

values smaller than zero may happen sometimes near $D_m \approx 0$ area. Such a minus *slope* of the D_m-curve may also be a feature of a short-range chaotic ASATI as confirmed by numerical experiments with artificial discrete chaotic data. The successive change of the *slope* around the chaotic period provides some quantitative view of the long-range correlation of the primary cosmic rays.

The time variation of the *slope* is calculated for the ASATIs obtained for several days including the chaotic ASATIs at every station. The calculation is performed with 150 or 200 ASATIs, shifting the first event by 2 or 4 to calculate the variation of the *slope*-value considering the difference of the event rate of each station. Typical results for the ASATIs observed at Nara Sangyo University, Okayama University and Hirosaki University are shown in figure 4. The time variation of the *slope* frequently shows quasi-periodic troughs around the right ascension 4 ± 1 h and 20 ± 1 h. This result corresponds well to the frequency distribution of the representative right ascension for the 99 cases of the chaotic ASATIs as shown in figure 3. The broadness of the troughs may correspond to the broadness of the peaks in figure 3, though the broadness depends on the number of ASATIs to determine the *slope* of the D_m-curve. Besides, the time variation of the *slope* sometimes has another trough around the right ascension 11 ± 1 h which is not covered by the broad troughs around the right ascension 4 h and 20 h. The quasi-periodic feature indicates that the long-range ASATIs for several days are also assumed not to be purely random.

3. Discussion

The origin of the chaos in cosmic rays is an enigma, since cosmic rays are supposed to be randomized after traversing long distances in more or less irregular galactic magnetic arms. High energies over 3×10^{14} eV exclude the possibility of an origin within the solar system, but the origin of chaotic features should not be too far and too old in the galactic disc.

Some simultaneous detections of chaotic ASATIs between two far-off stations indicate that the chaotic cosmic rays have a lateral extent. The quasi-periodicity mentioned also in section 2 is remarkable in the sense that the chaotic cosmic rays exhibit spatial structure of a few light days in extent, coming towards the earth from a certain direction. More than 90% of the total air showers are detected with a zenith angle less than 30°. So the chaotic cosmic rays could be observed continuously over 4 h if the chaotic cosmic rays arrive at the earth as a beam with high directronality. To detect the core part in the dominant 150 or 200 ASATIs, a part of the ASATIs is kept as originally and the rest of the ASATIs are replaced by random numbers before re-analysis of the whole of the ASATIs. If the chaotic feature, the constancy of the D_m-value, is not broken by the 10 times repeat execution of this process, then the part of the ASATIs kept as originally is considered to be the core part for the chaotic feature of the original ASATIs. The duration of the core part, detected with this process, is estimated to be more than 6 h at least, which is 2 h longer than 4 h mentioned above. Therefore, for three reasons, the simultaneous detection of the chaotic ASATIs between two far-off stations, the quasi-periodicity of the time variation of the *slope* of the D_m-curve and the long duration of the core part of the chaotic ASATIs, the chaotic cosmic rays could be assumed to arrive at the earth forming a fractal wave structure with a lateral extent of more than 15° at least.

Although spatial mapping is not yet possible for a chaotic group of cosmic rays owing to the scarcity of observing stations, a preliminary model of fractal density wave structure can be sketched as illustrated in figure 5. The sequence of intervals between the fractal waves, which have a self-similar structure, is observed as the chaotic ASATIs. According to this model, the quasi-periodic appearance of the chaotic ASATIs could be explained by the periodic observation of the chaotic ASATIs with the rotation of the earth. The wave front is

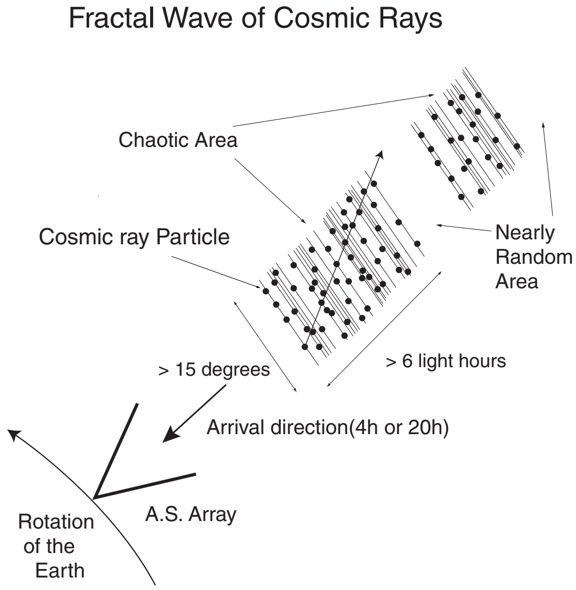

Figure 5. The illustration of a preliminary model of the fractal density wave structure of the chaotic cosmic rays. Small black dots represent the cosmic ray particles, and the straight line crossing the fractal wave pattern diagonally is the observational path of one station.

assumed to be perpendicular to the direction of propagation, although an oblique shock model may also be a possibility. The simultaneity of chaotic ASATIs observed at distant sites would indicate that the wave front should be approximately parallel to the earth's surface, and in that case the lateral extent of the wave pattern could be determined from the detection of the same chaotic group of cosmic rays at far distant sites. Worldwide observations are needed to detect the spatial extent of the wave pattern. Small black dots in figure 5 represent the cosmic ray particles, and the straight line crossing the wave pattern diagonally is the observational path of one station. If this picture is correct, the duration of chaotic series of ASATIs as long as a few hours, observed with a higher trigger rate, may give some indication about the lateral extent if the fractal wave structure has fine self-similarity. In the direction of incidence, a long belt of cosmic rays of several light days ($\sim 10^3$ AU) consisting of lateral stripes of fractal wave structures would be impinging against the solar system, as supposed from figure 5. The vertical depth of the fractal wave may be thin as the usual chaotic attracter, then the simultaneous observation of chaotic ASATIs between different stations should be difficult.

A question arises whether such a long string of fractal cosmic ray wave structure can be stable in travelling interstellar space? Is it bound by itself with a magnetic field like the radiation? Where and how is it formed? A speculation may be made for the source of a chaotic group of cosmic rays from the observed direction of cosmic rays and the intermittency of the chaos period. We may assume a nonlinear proton accelerator with a shock front in some supernova remnants [12] and a re-accelerator or an attenuater like a magnetic cloud which are located near the solar system as the source of the chaotic cosmic rays. The chaotic cosmic rays reaccelerated within several light years distance may arrive at the earth a few times a year keeping the chaos information narrow for more than six light hours after several Larmor rotations. The chaotic feature, the constancy of the D_m-value, is confirmed to be kept as originally in spite of the mixture of random noise up to 20% in the case of an artificial discrete chaos. If the re-accelerator releases the cosmic ray particles parallel to the direction of the regular magnetic field of the galactic plane, then the particles may arrive at the earth with minimum influence of irregular magnetic field or Larmor rotation. Each stripe of the fractal wave structure is considered to be the manifestation of nonlinear acceleration. Under this model, the chaotic ASATIs should be detected more frequently and the simultaneous detection of the chaotic ASATIs should happen more often if they did not suffer a mixing of random noise on the way from the accelerator.

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