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Nonlinear processes in cosmic-ray precursor of strong supernova shock: Maximum energy and average energy spectrum of accelerated particles

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Abstract

The instability in the cosmic-ray precursor of a supernova shock is studied. The level of turbulence in this region determines the maximum energy of accelerated particles. The consideration is not limited by the case of weak turbulence. It is assumed that the Kolmogorov type nonlinear wave interactions together with the ion-neutral collisions restrict the amplitude of random magnetic field. As a result, the maximum energy of accelerated particles strongly depends on the age of a SNR. The average spectrum of cosmic rays injected in the interstellar medium in the course of adiabatic SNR evolution takes the approximate form E^{-2} at energies larger than 10–30 GeV/nucleon with the maximum energy that is close to the position of the knee in cosmic-ray spectrum at 4×10^{15} eV. At an earlier stage of SNR evolution – the ejecta-dominated stage, the particles are accelerated to higher energies and have a rather steep power-law distribution. These results suggest that the knee may mark the transition from the ejecta-dominated to the adiabatic evolution of SNR shocks which accelerate cosmic rays.

Keywords: Cosmic rays; Supernova remnants; Shock waves; Turbulence

1. Introduction

The dependent on energy diffusion coefficient $D(E)$ determines the maximum energy that particles can gain in the process of acceleration. The condition of efficient acceleration is $D(E) \leqslant k u_{\mathrm{sh}} R_{\mathrm{sh}}$, where R_{sh} is the radius and u_{sh} is the velocity of spherical shock. Typically, the Pecklet number $k = 0.1$ in the free expansion stage and $k = 0.04$ in the adiabatic (Sedov) stage of SNR evolution, e.g., Berezhko et al. (1996). The Bohm value $D_{\mathrm{B}} = v r_g/3$ (v is the particle velocity, and $r_g = pc/ZeB$ is the particle Larmor radius) that is a lower bound of the diffusion along the magnetic field gives $E_{\max} = 2 \times 10^{14} Z (W_{51}/n_0)^{2/5}$ eV at the beginning of the Sedov stage when particles reach the highest energy. Here the SN burst with kinetic energy of ejecta $W_{\mathrm{sn}} = W_{51} \times 10^{51}$ erg in the gas with density $n_0\,\mathrm{cm}^{-3}$ and the interstellar magnetic field $B = B_0 = 5 \times 10^{-6}$ G is considered.

Analyzing the processes in very young SNRs when the shock velocity is high, $u_{\mathrm{sh}} \approx 10^4$ km/s, it was found by Bell and Lucek (2001) that the cosmic-ray streaming instability in the shock precursor is so strong that the amplified field $\delta B \geqslant 10^{-4}$ G far exceeds B_0. The maximum particle energy increases accordingly. The cosmic-ray streaming instability is less efficient as the shock velocity decreases with time and the nonlinear wave interactions reduce the level of turbulence at the late Sedov stage (Völk et al., 1988; Fedorenko, 1990). This leads to fast diffusion and decreases $E_{\max}$. The effect is aggravated by the possible wave damping on

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ion-neutral collisions (Bell, 1978; Drury et al., 1996). In the work Ptuskin and Zirakashvili (2003), we considered the acceleration of particles accompanied by the cosmic-ray streaming instability in a wide range of shock velocities. The level of magnetic field fluctuations was allowed to be arbitrarily high, and the rate of nonlinear wave interactions was assumed to correspond to the Kolmogorov type nonlinearity. The maximum energy of accelerated particles as a function of SNR age was found. In the present work, we continue that study and calculate the average spectrum of accelerated particles injected into the interstellar medium in the course of SNR evolution, see also Ptuskin and Zirakashvili (2004).

2. Maximum energy of accelerated particles

In the test particle approximation, the distribution of particles in momentum for high Mach number shocks has the canonical form $f(p) \sim p^{-4}$. In the case of efficient acceleration, the action of cosmic ray pressure on the shock structure causes nonlinear modification of the shock that changes the shape of particle spectrum making it flatter at relativistic energies. So, we assume that the distribution at the shock is of the form $f_0(p) \sim p^{-4+a}$ where $0 < a < 0.5$, and value $a = 0.3$ is used in the numerical estimates below. The normalization of function $f_0(p)$ is such that the integral $N = 4\pi \int \mathrm{d}pp^2 f_0(p)$ gives the number density of cosmic ray particles. We assume that the cosmic ray pressure at the shock is some fraction $\xi \leqslant 1$ of the upstream momentum flux entering the shock front, so that $P_{\mathrm{cr}} = \xi \rho u_{\mathrm{sh}}^2$. The typical value of $\xi \sim 0.5$ and the total compression ratio 7 were found in numerical simulations of strongly modified shocks by Berezhko et al. (1996).

The following steady-state equation determines the energy density $w(k)$ (k is the wave number) of turbulence amplified by the streaming instability in the cosmic-ray precursor upstream of the shock:

$$u \nabla w = 2(\Gamma_{\mathrm{cr}} - \Gamma_{\mathrm{l}} - \Gamma_{\mathrm{nl}})w. \tag{1}$$

Here the l.h.s. of the equation describes the advection of turbulence by supersonic gas flow with velocity $u(\mathbf{r})$. The terms on the r.h.s. of the equation describe, respectively, the wave amplification by cosmic rays, the linear damping of waves in background plasma and the nonlinear wave–wave interactions that limit the amplitude of turbulence. The Kolmogorov-type nonlinearity with the simplified expression $\Gamma_{\mathrm{nl}} = (2C_{\mathrm{K}})^{-3/2} V_{\mathrm{a}} k A(>k) \approx 0.05 V_{\mathrm{a}} k A(>k)$ at $C_{\mathrm{K}} = 3.6$ (as given by the numerical simulations of Verma et al. (1996)) is used in our calculations. Here V_a is the Alfven velocity, $A = \delta B / B_0$ is the wave amplitude. The wave-particle interaction is of resonant character and the resonance condition is $kr_g = 1$. The equations for Γ_{cr} and for the cosmic ray diffusion coefficient D used in our calculations generalize the standard equations (Berezinskii et al., 1990) derived in the case of weak random field. The details can be found in the paper Ptuskin and Zirakashvili (2003).

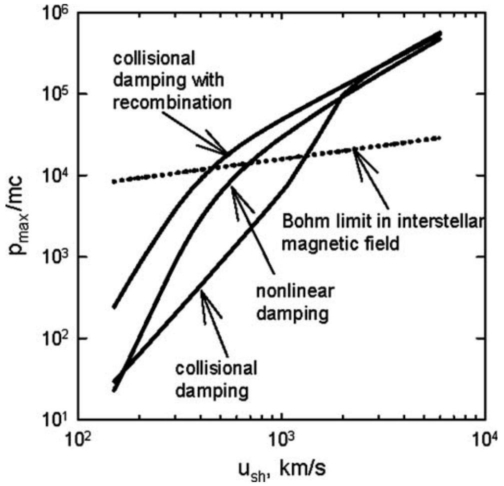

Fig. 1. The particle maximum momentum $p_{\max}$ of accelerated protons in units mc as a function of shock velocity u_{sh} calculated at $\xi = 0.5$, $k = 0.04$ for the $W_{\mathrm{sn}} = 10^{51}$ erg explosion in warm interstellar gas.

Fig. 1 illustrates the results of calculations of $E_{\max}$ at the Sedov stage of SNR evolution at $W_{\mathrm{sn}} = 10^{51}$ erg in the warm interstellar gas with temperature $T = 8 \times 10^3$ K and average density $n_0 = 0.4\ \mathrm{cm}^{-3}$. Three solid lines correspond to three cases of wave dissipation considered separately: nonlinear wave interactions; damping by ion-neutral collisions at constant gas density; damping by ion-neutral collisions when the diffuse neutral gas restores its density after complete ionization by the radiation from SN burst. The maximum energy of protons accelerated by SN shocks at the early Sedov stage is close to 5×10^{14} eV (that exceeds the Bohm limit) and decreases to about 10^{10} eV at the end of the Sedov stage (that is less than the Bohm limit). In particular, the particle energy is less than 10^{13} eV for $t > 3 \times 10^3$ yr and this may explain the absence of TeV gamma-ray signals from many not young SNRs (see Buckley et al., 1998) where the very high energy gamma-rays could be produced through $\pi^0 \to 2\gamma$ decays if sufficiently energetic cosmic rays were present.

The highest particle energy estimated as $E_{\max} = 2 \times 10^{16} Z (u_{\mathrm{sh}}/10^4\ \mathrm{km/s})^2 \xi M_{\mathrm{ej}}^{1/3} n^{1/6}$ eV can be reached at the end of the free expansion stage (here M_{ej} is the mass of ejecta in solar masses and the value $k = 0.1$ is accepted).

3. Average spectrum of injected cosmic rays

At a given SNR age t, the cosmic rays are accelerated up to a maximum momentum $p_{\max}(t)$. Also, particles with $p > p_{\max}(t)$ cannot be confined in the precursor of the shock even if they were accelerated earlier. Thus particles accelerated to the maximum energy escape from a SNR (see also Berezhko and Krymsky, 1988). Let us estimate the flux of these run-away particles. We consider the simplified approach and assume that $D \ll u_{\mathrm{sh}} R_{\mathrm{sh}}$ at $p < p_{\max}$ and $D \gg u_{\mathrm{sh}} R_{\mathrm{sh}}$ at $p > p_{\max}$. The spectrum of the accelerated particles in this case has a very steep cutoff at $p = p_{\max}$ and the spectrum of run-away particles is approximated by a delta-function which can be found from the solution of the diffusion-convection transport equation for particle distribution function downstream of the spherical blast wave shock produced by supernova explosion, see Ptuskin and Zirakashvili (2004) for details:

$$\begin{aligned} q(p,t) = {} & 4\pi\delta(p-p_{\max})\Bigg[\frac{1}{3}\left(1-\frac{1}{\sigma}-\frac{\xi}{2}\right)R_{\rm sh}^2 u_{\rm sh} p f_0(p) \\ & +\left(-\frac{\partial p_{\max}}{\partial t}-\frac{\sigma-1}{\sigma}p_{\max}\frac{u_{\rm sh}}{R_{\rm sh}}\right) \\ & \times\int_0^t \frac{dt_1}{\sigma}R_{\rm sh}^2(t_1)u_{\rm sh}(t_1)f_0\left(p\left(\frac{R_{\rm sh}(t)}{R_{\rm sh}(t_1)}\right)^{1-\frac{1}{\sigma}},t_1\right)\left(\frac{R_{\rm sh}(t)}{R_{\rm sh}(t_1)}\right)^{3-\frac{3}{\sigma}}. \end{aligned} \quad (2)$$

The expression in brackets in front of the integral in Eq. (2) should be positive, which means that adiabatically in a process of SNR expansion the particles lose energy more slowly than the maximum energy decreases. For the opposite sign, the adiabatic losses are faster than the decrease of maximum energy and the particles do not run away from downstream of the shock. The integral $4\pi\int dp p^2 q(p,t)$ has dimensions number of particles per unit time. The compression ratio of strongly modified shock is $\sigma \approx 7$.

The average source power $Q(p)$ of run-away cosmic rays per unit volume in the galactic disk is obtained by integrating $q(p,t)$ with respect to t and by averaging over many SN explosions: $Q(p) = v_{\rm sn}\int dtq\ (p,t)$, where $v_{\rm sn}$ is the average frequency of SN explosions per unit volume of the galactic disk.

Fig. 2 presents the results of calculation of the average proton source spectrum for the acceleration in two types of supernovae. The first is the typical Type II SN which goes through the Red Super Giant phase before explosion so that the shock propagates through the dense low-velocity wind before entering the hot low-density bubble produced by the star while on the main sequence and finally vanishing in the interstellar medium, see, e.g., Lozinskaya (1992). The nonuniform distribution of gas density along the shock propagation path causes the irregularities in the average source spectrum. The second, probably a factor of 4 less frequent in the Galaxy, is the Type Ia SN which explodes in the uniform warm interstellar medium. The calculated average source spectrum of protons is close to $Q(p) \sim p^{-4}$ at energies less than about 6×10^{15} eV for Type II SN, and about 3×10^{15} eV for Type Ia SN. This part of the spectra is formed during the adiabatic (Sedov) evolution of a SNR. In both cases, the average source spectrum $Q(p)$ has a break at high energy and it may explain the knee in the spectrum of galactic cosmic rays observed at about 4×10^{15} eV. The high energy part of the spectrum is formed during the acceleration at the ejecta-dominated stage of the SNR evolution.

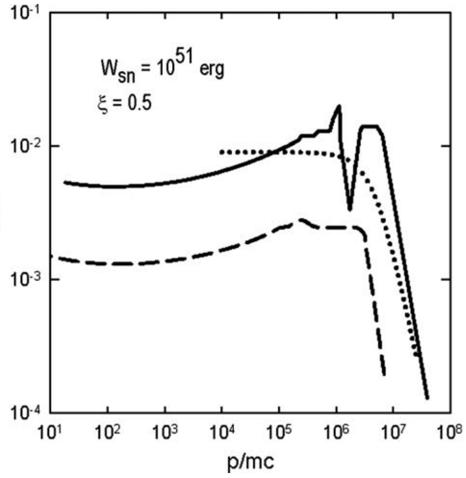

Fig. 2. The solid line shows the average source spectrum $Q(p)p^4c$ (given in units $(\xi/0.5)v_{\rm sn}W_{\rm sn}$ per steradian) for protons released into the interstellar medium during SNR evolution after SNII explosion in the wind of a RSG progenitor star. The Pecklet number is changing from $k = 0.04$ at low energies to $k = 0.1$ at high energies. The dashed line presents the case of a SNIa explosion in a uniform interstellar gas; the average source spectrum is multiplied by 1/4. The dotted line shows the shape of the proton source spectrum used by Hörandel (2003) to fit the data of KASCADE experiment on the spectrum and composition of very high energy cosmic rays.

The overall energy spectrum of cosmic rays observed at the Earth above 10^{10} eV where it is not significantly influenced by the solar wind modulation is well described by a power law with the prominent feature, the “knee”, at 4×10^{15} eV, the second knee at 3×10^{17}–10^{18} eV, and the flat supposedly extragalactic component at energies more than about 3×10^{18} eV. The power law dependence of cosmic ray intensity on energy $I(E) = p^2 f(p) \sim E^{-\gamma-a}(\approx E^{-2.7}$ at energies 10^{10}–10^{15} eV) combines the energy dependent source term $q(E) = p^2Q(p) \sim E^{-\gamma}$ and the energy-dependent time of cosmic-ray leakage from the Galaxy $T \sim E^{-a}$. The leakage exponent is $a = 0.3$–0.6 depending on details of the propagation model in the Galaxy, see Jones et al. (2001). The source spectrum exponent should be then $\gamma = 2.1$–2.4 in a fair agreement with the presented above results.

According to the theory, the breaks and cutoffs in the spectra of ions with different charges should occur at the same magnetic rigidity, i.e., at the same ratio E/Z for the ultrarelativistic nuclei. The data of recent KASCADE experiment for the most abundant groups of nuclei (protons, helium, CNO group, and the iron group nuclei) are, in general, consistent with this concept. According to Hörandel (2003), the fit to the observations is reached if an individual constituent ion spectrum has a gradual steepening by $\Delta\gamma \sim 2$ at approximately $4\times10^{15}Z$ eV that is in agreement with our model prediction.

The serious problem of data interpretation exists with the second knee in the cosmic ray spectrum. The natural assumption that each individual ion has only one knee at the same ratio E/Z for all species and that the knee in the spectrum of iron nuclei ($Z = 26$) expected then at about 10^{17} eV would explain the second knee in the all-particle spectrum does not agree with the observed position of the second knee at about 5×10^{17} eV. The way out was suggested in the work of Sveshnikova (2004) where the dispersion of parameters of SN explosions was taken into account in the calculations of the knee position and the maximum particle energy. It leads to the widening of the energy interval between two knees in the overall all-particle spectrum. This analysis should be supplemented by the account of different chemical composition of the progenitor star winds that determines the composition of accelerated cosmic rays (Silberberg et al., 1990). The scaling of the knee position in our model is $p_{\text{knee}} \propto Zk\xi W_{\text{sn}}\sqrt{\dot{M}}M_{\text{ej}}^{-1}u_{w}^{-1}$ for explosion in the stellar wind and $p_{\text{knee}} \propto Zk\xi W_{\text{sn}}M_{\text{ej}}^{-2/3}n_0^{1/6}$ for explosion in the uniform interstellar medium.

4. Discussion and conclusion

The accounting for nonlinear effects which accompany the cosmic ray streaming instability raises the maximum energy of accelerated particles in very young SNRs above the standard Bohm limit calculated for the interstellar magnetic filed strength by about two orders of magnitude up to $\sim 10^{17}Z$ eV (with the corresponding increase of random magnetic field up to $\sim 10^{-3}$ G). The nonlinear wave dissipation can also considerably reduce the maximum energy of particles that are present inside SNRs to 10–30 GeV at the end of Sedov stage. These results give a clue to understanding why very young SNRs can be efficient accelerators of very high energy particles (as evidenced by their TeV emission and the synchrotron X-ray emission) but the SNRs with ages more than about 3×10^3 yr are not bright in very high energy γ-rays.

In the present work, we studied the effect of a strong time dependence of the maximum particle momentum $p_{\max}(t)$ on the average spectrum of cosmic rays injected into interstellar space from many supernova remnants over their lifetime. The instantaneous cosmic ray spectrum at a strongly modified shock is flat and the particle energy density is mainly determined by the particles with maximum momentum. The instantaneous source spectrum of the run-away particles is close to a delta function. At the same time, the assumption that a constant fraction ξ of incoming gas momentum flux goes into the cosmic ray pressure at the shock, and the fact that the supernova remnant evolution is adiabatic leads to an average source spectrum for ultrarelativistic particles from the ensemble of SNRs that is close to the universal form $Q \sim p^{-4}$ from energies about 30 GeV/n up to the knee position in the observed cosmic ray spectrum. This source spectrum is consistent with the empirical model of cosmic ray propagation in the Galaxy. The acceleration at the preceding ejecta-dominated stage of SNR evolution provides the steep power-law tail in the particle distribution at higher energies up to about 10^{18} eV (if the iron nuclei dominate at these energies). The knee in the observed energy spectrum of cosmic rays at 4×10^{15} eV is explained in our model by the transition from the ejecta-dominate stage to the adiabatic stage of SNR shock evolution.

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