ACCELERATION OF COSMIC RAYS BY SHOCK WAVES

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1. Introduction

The problem of the origin of galactic cosmic rays is a particularly difficult one despite the fact that rather detailed measurements of the properties of cosmic rays can be made, at least in the vicinity of the Sun. The current situation has been well reviewed by Lingenfelter (1) who points out that there are several linked problems to be solved, namely the question of sources and acceleration mechanisms, propagation within the galaxy, escape from the galaxy and of course solar modulation which affects the interpretation of the observations, especially below $\sim$ 1 GeV/nuc. It is usually supposed that these problems can be treated separately, so that the sources (perhaps supernovae, pulsars, black holes, flare stars, etc.) simply provide cosmic rays with given elemental and isotopic abundances and given spectra, which then propagate independently by diffusion through the interstellar medium, producing secondaries and perhaps losing energy as they do so until they eventually leave the galaxy by some means, which is usually described in terms of a "free escape" boundary condition to the diffusion equations.

In recent years there has been renewed interest in the possibility that the acceleration of cosmic rays should occur, not in discrete sources, but in the diffuse interstellar medium, as a consequence of shock waves associated with supernova remnants (2-5). Since the supernova remnants concerned are rather large and indeed tend to dominate the whole interstellar medium (6) it is becoming clear that the problems of acceleration and propagation of cosmic rays cannot be so easily separated. A further difficulty is concerned with the escape of cosmic rays from the galaxy which may be associated with a galactic wind (7-9) which is partly driven by cosmic ray pressure and therefore not an independent process. These complexities give added interest and significance to the role of cosmic rays in the dynamics of the interstellar medium but of course also make the traditional problems of cosmic ray physics much more difficult to treat.

We will attempt here to review the current status of investigations into various aspects of the problem of shock acceleration of cosmic rays. Due to space limitations and the uneven nature of our progress, not every aspect can be treated in detail and many of the problems associated with non-linear effects (section 10) are left to a later review. The reader is referred to other review articles for further information (10-14) and also to related work on the acceleration of particles at the Earth's bow shock (15 and references therein).

The idea that collision-free shocks exist was first put forward by Gold in 1953 as an explanation for the short duration of the sudden commencements of geomagnetic storms (16). Disucssions of the processes by which such shocks can accelerate energetic particles have until recently been rather fragmentary with numerous authors proceeding along roughly the same paths quite independently of each other.

The first observations of such an effect occurring in interplanetary space was made in 1959 by Dorman et al. (17-19) who noted that small (∿ 1%) increases of cosmic ray intensity sometimes occur before the sudden commencement of a geomagnetic storm which is usually followed by a much larger (∿ 10%) "Forbush" decrease of intensity. Large (∿ 100%) enhancements of solar energetic particle fluxes prior to geomagnetic storm sudden commencements were discovered by Reid and Axford (20,21) in 1962, using high latitude riometer observations. Spacecraft observations of such "pre-SC energetic particle enhancements" were reported by Bryant et al. (22) in 1962. Since that time very detailed observations have become available of intensity increases associated with solar flare shock waves (23-25), corotating interaction regions (26-28) and the magnetospheric bow shocks of the Earth (15,29-31) and Jupiter (32,33). Two classes of event can be distinguished, namely (a) smooth, quasi-exponential increases occurring in front of a shock wave, lasting many hours in the case of interplanetary shocks, and (b) short-lived intensity bursts ("shock spikes") lasting typically tens of minutes and exhibiting rather high field-aligned anisotropies (34-36). In addition, there is clear evidence for strong non-thermal heating of the shocked plasma (37,38) and for escaping particle beams upstream from shocks which may represent ions and electrons which have been directly accelerated from the plasma itself (15,39,40). It is an advantage of the shock acceleration theory for cosmic rays that, in contrast to many other theories, the mechanisms involved can be observed directly and in great detail in interplanetary space.

The first attempts to deal with theoretical aspects of shock acceleration were made by Dorman and Freidman (41) and Shabansky (42) on the one hand, Parker (43), Wentzel (44,45) and Hudson (46) on the other, all considering scatter-free acceleration as reviewed in section 4. Hoyle (47) used the Rankine-Hugoniot conditions for a shock, including the cosmic ray pressure and enthalpy and assuming that the background gas behaves isotropically to obtain a very interesting result which is a special case of the non-linear theory reviewed in section 10. The significance of scattering in the medium on either side of the shock was noted independently by several authors, in particular Schatzman (48), Jokipii and Davis (49,50) and Van Allen and Ness (51). Little progress could be made in this direction, however, until suitable transport equations became available and the transition conditions for the cosmic ray distribution function at the shock were understood (52-56). Numerous solutions of these equations with the appropriate transition conditions (56,11) were obtained by Fisk (57-59) but with the assumption that the energetic particle spectrum has a specific power law form and overlooking a particular solution of the equations. A numerical solution of the transport equations simulating a solar flare event with a propagating interplanetary shock wave was given by Morfill and Scholer, taking into account shock reflection and second order Fermi acceleration (59). Independently, and more or less simultaneously, Krimsky (2), Axford, Leer and Skadron (3), Bell (4) and Blandford and Ostriker (5) found the solution of the steady, one-dimensional problem (see section 5) and noted its obvious applications to the cosmic ray acceleration problem, especially in terms of the efficiency of the process, the possibility of achieving power law spectra and the prevalence of shock waves in the interstellar medium.

A summary of the observed properties of galactic cosmic rays and the inferences which can be drawn from them is given in section 2. It should be remembered that some of these inferences are based on the most simple-minded view of cosmic ray propagation in the galaxy, ignoring the complications mentioned above, and hence may eventually have to be revised. Furthermore, the observations are not always entirely consistent with each other and it has been necessary to choose particular values for various parameters which may also require revision. These remarks also apply to section 3 where we review the basic properties of the interstellar medium with the aim of providing representative values of various important parameters as a basis for subsequent discussion. It is already emphasized at this stage that cosmic rays must play an important role in the dynamics of the interstellar medium since, if the medium is sufficiently strongly coupled to the cosmic rays to account for their acceleration, then the reverse must be true and the cosmic rays therefore affect the dynamics of the medium itself. This has important implications for the behaviour of supernova remnants, the nature of supersonic turbulence in the interstellar medium and for the formation of a galactic wind.

The basic elements of scatter-free acceleration of energetic particles by shock waves are outlined in section 4. We emphasize that although some interesting and significant phenomena can result from simple reflection and transmission of particles at shock waves without additional scattering, this does not provide a satisfactory basis for an explanation of galactic cosmic ray acceleration. The essential features of the more important mechanism in which scattering is included are outlined in section 5. It is shown that for a steady, plane shock with given scattering properties upstream and downstream (i.e. given cosmic ray diffusion coefficients) cosmic rays are efficiently and irreversibly accelerated in such a manner that power law energy/momentum spectra are a natural consequence. The important modifications associated with time dependence and losses due to interaction with the background medium are described in section 6 and the effects of non-planar geometry in section 7. These additional effects, which at some stage must be considered in any attempt to model the real situation, lead in general to spectra which are not power-law in form.

The possibility that galactic cosmic rays are accelerated by supernova shocks is discussed in section 8. It is evident that there are many attractive and plausible features of such a scheme. However, there are some conceptual difficulties notably with regard to the role played by cosmic rays in the dynamics of the supernova remnants and the means by which the cosmic rays and the remnants eventually merge with the interstellar medium and flow into the galactic halo from which they eventually escape. Nevertheless, with some reasonable but perhaps not totally justifiable assumptions, Blandford and Ostriker (60,61) have been able to construct a model which accounts for the observed spectra of cosmic ray primaries rather well. It should be noted however, that this is again based on the assumption that the cosmic rays do not affect the dynamics of the interstellar medium so that the model goes only one step beyond the familiar "leaky box" approach by including a consistent source mechanism.

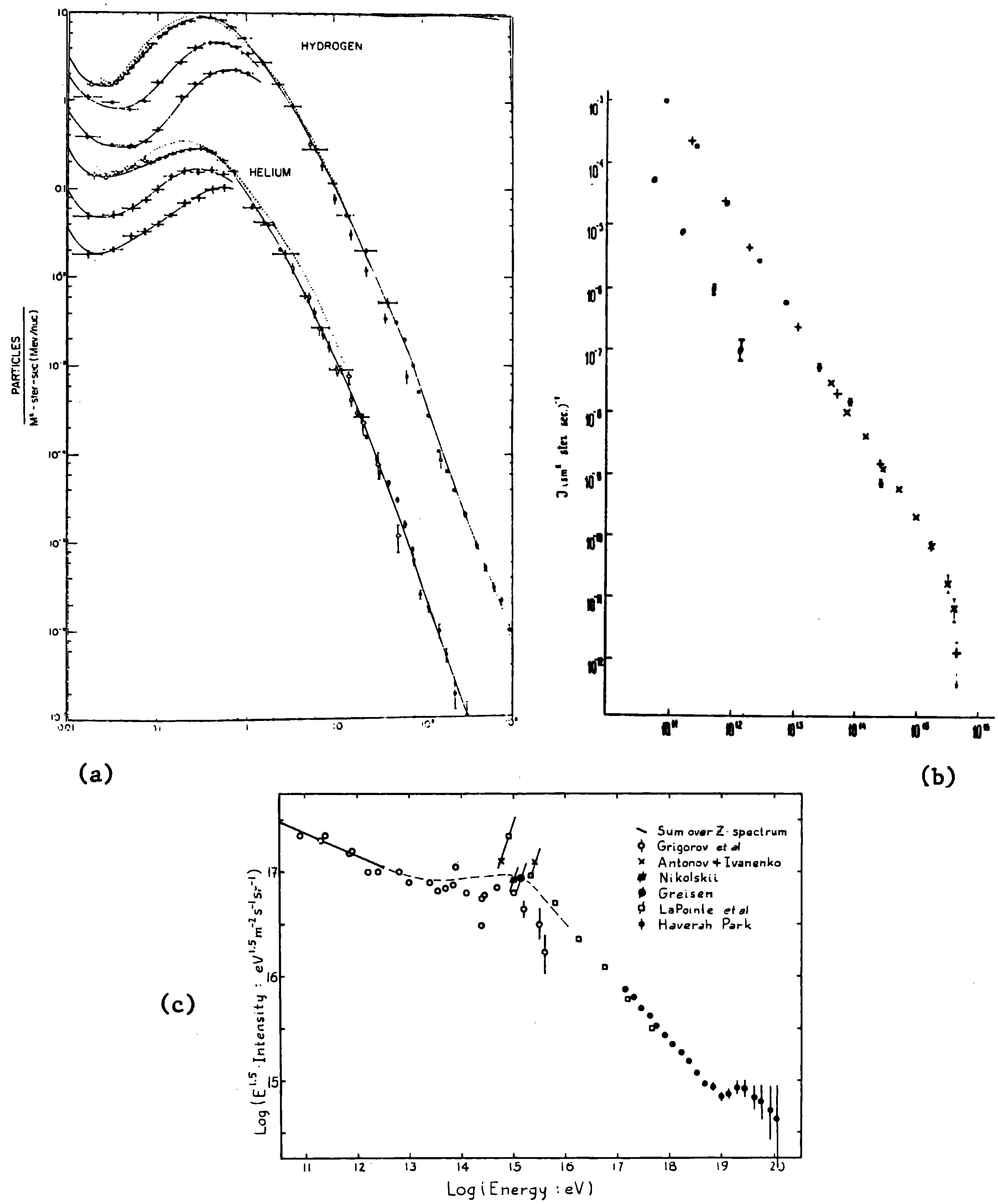
One of the difficulties encountered by the shock acceleration theory is that it requires that there be sufficient scattering in the interstellar medium by magnetic field irregularities for the acceleration to be efficient up to energies exceeding certainly 10^3 GeV/nuc and hopefully up to 10^5-10^6 GeV/nuc where there is some evidence that changes in the spectrum occur (62-64) and the anisotropy begins to increase (65,66). This problem, which has been emphasized in particular by Ginzburg and Ptuskin (67) and Cesarsky and Lagage (68) and is discussed in section 9, leads one rather quickly to conclude that a completely non-linear treatment is necessary. This must take into account the generation, amplification and damping of hydromagnetic waves in a medium with strong supersonic turbulence and streaming cosmic ray fluxes (4,69-73). A brief outline of the present state of this aspect of the shock problem is given in section 10.

2. Galactic Cosmic Rays

The differential spectrum of primary galactic cosmic ray protons has the form $j(T) \propto T^{-\mu_1}$, $\mu_1 \sim 2.65$, in the kinetic energy range $10 \leq T \leq 10^6$ GeV (Figure 1). At lower energies ($1 \leq T \leq 10$ GeV) the spectrum is somewhat flatter and is noticeably affected by solar modulation. The spectrum becomes slightly steeper in the region above 10^6 GeV and there are possibly further changes of slope at higher energies (62-64). The break in the spectrum at $\sim 10^6$ GeV, together with the observation that the anisotropy increases from very low values (10^{-2}-10^{-1}%) to high values (1-100%) in the range 10^6-10^{12} GeV suggests that extragalactic particles become progressively more prominent at higher energies (65,66). We know essentially nothing about the spectrum at energies $T \ll 1$ GeV due to the overwhelming effects of solar modulation, although it is often assumed for convenience (but without real justification) that it has the form of a power law in total energy, namely $(T + T_o)^{-\mu_1}$, where T_o is the rest mass energy (see Figure 2).

The spectra of other primary species appear to be roughly the same as that of protons when expressed in terms of energy per nucleon (64,75, 76), but again nothing can be said about the forms of the unmodulated spectra at low energies. The relative abundances of the primaries correspond approximately to the solar and local galactic abundances deduced by Cameron and others as far as the heavier elements are concerned (77). However, the lighter elements appear to be less efficiently accelerated so that H, He and C, N, O, for example, are suppressed by factors of order 30, 20 and 2-5, respectively, with respect to the relative abundances of Si, Ca, Fe, etc. (78-81). There is also a tendency for the neutron-rich isotopes of Ne, Mg, etc. to be somewhat overabundant in the cosmic radiation in comparison with solar system abundances (80,82) which may well reflect a genuine difference between the compositions of the Sun (representing the interstellar medium $\sim 4\times10^9$ years ago) and the cosmic ray source material (possibly representing the interstellar medium during the past 10^{7-8} years) (83,84). It is interesting and possibly significant that the relative abundances of elements in the galactic cosmic ray source are rather similar to those of solar energetic particles (80,85).

If we relate the differential number density $U_i(T)$ of a species i



<u>Fig. 1.</u> (a) Differential energy/nucleon spectra of protons and alpha-particles for various phases of the solar cycle (compiled by Webber (215)).
(b) Satellite measurements of the integral energy/nucleon spectra of protons and alpha-particles (Grigorov et al. (216, 217)).
(c) Integral total energy spectra obtained by a variety of techniques in the range 10^{12}–10^{20} eV (62).

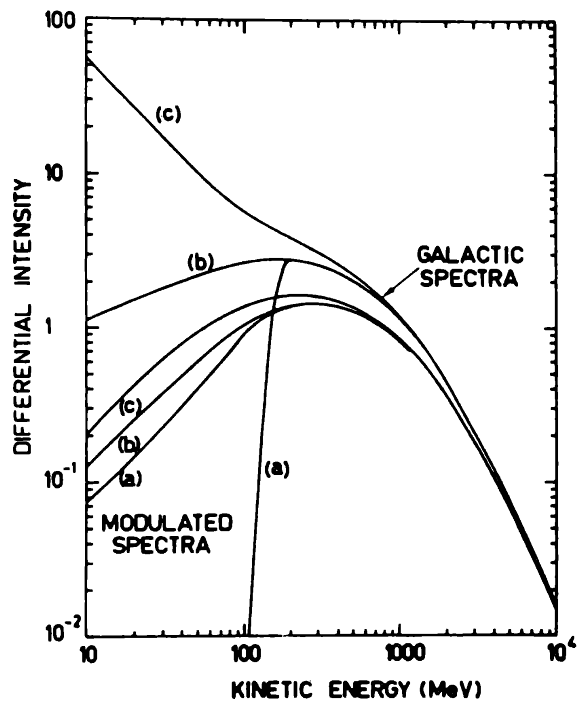


Fig. 2. Model calculations of the modulated spectra of galactic cosmic ray protons observed at the Earth, corresponding to three different assumed forms of the unmodulated spectrum (74). The modulation parameter is ϕ = 0.140 GV. Note the insensitivity of the unmodulated spectrum below $\sim$ 200 MeV to the form of the modulated spectrum, indicating the difficulty in interpreting observations in this range and of determining the unmodulated spectrum at low energies.

to its differential intensity by means of $j_i(T) = vU_i(T)/4\pi$ where v is the particle speed corresponding to a kinetic energy per nucleon of T, then the total (internal) energy density and pressure of the cosmic radiation, W_c and p_c respectively, can be expressed as

$$W_c = \sum_i \int_o^\infty A_i T\ U_i(T)dT\ , \quad p_c = \sum_i \int_o^\infty \frac{1}{3}\alpha\ A_i T\ U_i(T)dT \qquad (2.1)$$

where A_i is the atomic mass of the species i and $\alpha = (T + 2T_o)/(T + T_o)$. From observations made at solar minimum and with $\alpha \sim 1$, it is deduced that $W_c \sim 1.6\times10^{-12}$erg/cm^3 and $p_c \sim 0.6\times10^{-12}$dyn/cm^2 (86). These estimates may well be too low by a small factor as result of a neglect of residual modulation at solar minimum and the possible existence of substantial undetected fluxes in the range $T \lesssim 1$ GeV/nucleon (see Figure 2). The total energy density (enthalpy) of the observed cosmic rays is $H_c = W_c + p_c \sim 2.2\times10^{-12}$erg/cm^3.

The cosmic ray electrons are evidently primary particles but have a somewhat softer spectrum than that of protons, presumably as a result of inverse Compton and synchrotron losses. The bending of the measured electron spectrum at high energies is consistent with an age of the order of 2×10^7 years (87-90). Assuming diffusive propagation, it can be shown that the cosmic ray electrons we observe directly must originate within $\sim$ 1 kpc of the Sun (89). At a given kinetic energy the electron flux is $\sim$ 100 times smaller than that of protons (90), but the integral fluxes may not differ so much if it should be the case that there are many more electrons in the range below 1 GeV.

Secondary particles produced by nuclear interactions with the interstellar medium usually have a differential spectrum of the form $j(T) \propto T^{-\mu_2}$, $\mu_2 \sim 3$, in the range $3 \lesssim T \lesssim 10^2$ GeV/nucleon (76,91-93). Since the secondary production spectrum should be approximately the same as that of the primaries, the difference between the spectral indices μ_1 and μ_2 suggests that the residence time in the galaxy (τ) depends on energy, with the more energetic particles escaping more easily:

$$\tau = \tau_o(T/T_1)^{-0.35}\ , \quad T > T_1 \sim 3 \text{ GeV/nuc} \qquad (2.2)$$

where τ_o is the residence time for $T = T_1$ (94,95). The fact that $\mu_2 > \mu_1$ is significant in that it implies that cosmic rays cannot be the result of a general and continuous acceleration process operating thoughout the interstellar medium since this would produce a secondary spectrum flatter than that of the primaries, i.e. $\mu_2 < \mu_1$ (96-99). Furthermore, it suggests that the primary source spectrum has the form $j(T) \sim T^{-2.3}$ for $10 \le T \le 10^6$ GeV.

The observed fluxes of secondaries indicate that for primaries with $T < T_1$, $\bar{\rho}v\tau \sim 7$ v/c gm/cm^2, where $\bar{\rho}$ is the mean density of the interstellar medium in the volume occupied by cosmic rays while they are retained by the galaxy (94,95,100-102). Measurements of the flux of the radioactive isotope Be^{10} suggest that $\bar{\rho}$ is equivalent to $\sim$ 0.22 H atom/cm^3 and $\tau_o \sim 2\times10^7$ years (103-105). Since the mean density of matter within $\pm$ 100 pc of the central plane of the galaxy is $\sim$ 0.4 H atom/cm^3

(106), the cosmic ray storage volume in the above-mentioned sense must have a half thickness of $\sim$ 200 pc and a total volume $V_g \sim 8\times10^{66}$ cm^3 (assuming a disc of 15 kpc radius). The mean power of the cosmic ray source must therefore be $V_g(W_c+p_c)/\tau_o \sim 4\times10^{40}$erg/sec. It should be noted that these conclusions are based on the assumption that no significant post-acceleration of secondary particles occurs; this would have the effect of reducing the primary path length.

It is evident from the inhomogeneous distributions of the non-thermal radio emission associated with cosmic ray electrons (107) and the diffuse gamma-ray emission due to interactions of cosmic ray nuclei with the interstellar medium (108), that the distribution of cosmic rays in the galaxy is not uniform. This has implications for the cosmic ray sources since they should presumably be in some way associated with the regions in which the cosmic ray intensity is higher than average. Accordingly supernova remnants and the vicinities of OB associations are possible source candidates. There is evidence also that the cosmic ray intensity is noticeably higher towards the centre of the galaxy (109, 110). This may indicate that a peculiar source exists at the centre, or that there is a higher concentration of more normal sources, or simply that conditions are such that a higher cosmic ray pressure can be maintained in this region (109,111). It seems unlikely, however, that the bulk of the cosmic rays we observe can come from such a distant source since the electrons we observe directly probably originate from within $\sim$ 1 kpc of the Sun and there are no strong reasons for believing that the nuclear component behaves differently.

A consequence of the inhomogeneity of the distribution of cosmic rays is that as a result of the peculiar motions of the Sun and the sources and the time variations of the latter, the Sun must at times find itself in regions of higher than average cosmic ray intensity. The fact that the observed intensity does not seem to have fluctuated by more than $\pm$ 30% during the past 10^5-10^6 years and has remained roughly constant (to within a factor $\sim$ 2) over the last 10^8 years therefore places constraints on the sizes and number of the source regions (112, 113).

On the basis of the above considerations it appears that only supernovae and their remnants can provide enough power to account for the galactic cosmic rays (86,12). The overall efficiency of the acceleration process is of the order of 10% but since energy exchange is not a one-way process (i.e. cosmic rays can perform work on the medium in which they find themselves) the acceleration mechanism(s) must be capable of working at close to 100% efficiency at least temporarily. Moreover, we must rely on sources existing within $\sim$ 1kpc of the Sun to provide the local cosmic rays and the main acceleration must occur within a relatively small fraction (<< 10%) of space in order to leave the spectra of secondaries largely unaffected and to avoid a high probability for the observation of large temporal variations at the Sun. The observed small anisotropy for $T \lesssim 10^6$ GeV suggests that many sources may be involved and/or that there is a considerable amount of scattering of cosmic rays in the interstellar medium.

In order to achieve a single power law in $10 \leq T \leq 10^6$ GeV, it seems that a dominant acceleration process must exist. Since the source composition is rather normal apart from the suppression of lighter elements such as H, He, CNO, etc., relative to Si, Fe, etc., it is necessary that the particles accelerated should be extracted from a plasma of normal composition, having in particular no significant dust component which would tend to reduce the supply of non-volatile elements. Finally, the acceleration must be relatively prompt in the sense that it takes particles from thermal (plasma) to cosmic ray energies in a time which is short compared with τ and with the minimum time scale of any loss mechanism that can occur at intermediate energies. Otherwise the abundance distribution would be distorted in favour of H, He, etc. relative to Si, Fe, etc. in disagreement with the observations (114).

From many points of view it appears that shock acceleration in the hot component of the interstellar medium should be considered a promising source for galactic cosmic rays because (a) the mechanism is efficient, (b) supernova shocks can provide the necessary power, (c) the hot interstellar medium combines normal composition with sufficiently low densities to minimize any effects of energy losses, (d) power law energy spectra can be achieved under suitable circumstances, and (e) the shock waves which determine the spectrum occupy only a small fraction (<< 10%) of interstellar space.

3. The Interstellar Medium

It is sufficient for our purposes here to assume a rather simple model for the interstellar medium and its extension into the galactic halo. We assume that the interstellar medium can be divided into three phases: hot, intermediate and cold, of which the hot component occupies most of space and the cold component contains most of the mass (6,115).

The hot interstellar medium (HISM) is the result of supernova explosions and stellar winds and may be regarded as being in a state of supersonic turbulence (116). It should contain little dust, so that it is a plasma of essentially "normal" composition which is suitable as a source region for galactic cosmic rays. For the purposes of further discussion we assume representative values of the number density (n), temperature (T_g) and magnetic field (B) to be $n = 3\times10^{-3}$ cm^3, $T_g = 7\times10^5$ K and $B \leq 1$-3 μGauss. The HISM occupies most of the cosmic ray containment volume V_g and hence involves a mass of the order of 2×10^7 $M_\odot$.

The cold component has densities greater than 1/cm^3, temperatures less than 10^4 K and magnetic field strengths of the order of 3 μGauss. The gas contains much dust and hence does not have normal composition, being deficient in non-volatile elements in particular. The volume of space occupied by the cold component does not exceed 0.2 V_g and is probably substantially smaller. The total mass of the cold component is of the order of 2×10^9 $M_\odot$.

It is difficult to estimate the amount of material involved in the intermediate phase which comprises the denser HII regions as well as the transition zones between the hot and cold components where energy transfer by heat conduction and radiation plays a role. The importance of this component in controlling the dynamics of the HISM depends somewhat critically on whether heat conduction is impeded by magnetic fields and microscopic plasma instabilities (117,118).

The supernova rate in the outer parts of the galaxy near the Sun has been estimated to be of the order of 1/10-1/30 per year (119). Assuming that each explosion involves the release of 10^{51} ergs, the average supernova power is $\sim 10^{42}$-3×10^{42} erg/sec which is sufficient to account for the cosmic ray source power requirement. Other possibly significant sources, which like supernovae tend to be associated with young hot stars, are stellar winds ($\lesssim 10^{41}$ erg/sec) and HII regions ($\lesssim 10^{40}$ erg/sec) (12,120). The mass injection rate from supernovae and stellar winds is $\sim 2M_{\odot}$/year, most of which is likely to go directly into the HISM. In any case, the turn-over time for the interstellar medium appears to be at most of the order of 10^9 years, which is less than the age of the solar system and therefore allows some change in average composition to occur. It is conceivable that the turnover time for the HISM could be much smaller ($\sim 10^7$ years) if the gas is injected directly from supernovae and stellar winds and leaves the galaxy in the form of a galactic wind.

The pressures and internal energies associated with the plasma, cosmic rays and magnetic fields in the HISM are as follows:

$$p_g \sim 1.1\times10^{-12},\quad p_c \sim 0.6\times10^{-12},\quad p_M \sim 0.04\text{–}0.4\times10^{-12}\ \text{dyn/cm}^2\ ;$$

$$W_g = 1.6\times10^{-12},\quad W_c = 1.8\times10^{-12},\quad W_M = 0.04.\text{–}0.4\times10^{-12}\ \text{ergs/cm}^3.$$

The speed of sound (c_s) in the HISM, which is the minimum shock speed, is estimated as follows:

$$c_s^2 = \gamma p_g/\rho + \gamma_c p_c/\rho + B^2/4\pi\rho = c_g^2 + c_c^2 + c_A^2\ , \qquad (3.1)$$

$$c_g \sim 165\ ,\quad c_c \sim 110\ ,\quad c_A \sim 35\text{–}110\ ,\quad c_s \sim 200\text{–}230\ \ \text{km/sec.}$$

Here $\gamma = 5/3$ and $\gamma_c = 1 + \bar{\alpha}/3$ are the specific heat ratios for the plasma and cosmic ray gas, respectively.

Note that the β of the plasma is large, especially if the contribution of cosmic rays is included: $\beta_g = p_g/p_M \sim 3\text{–}30$; $\beta = (p_c + p_g)/p_M \sim 4\text{–}40$. The total energy density in the HISM is $H = \Sigma p + \Sigma W \sim 6\times10^{-12}$erg/cm^3. Hence the total energy per unit mass is 10^{15}ergs/gm, corresponding to a speed of ~ 450 km/sec, which exceeds the speed of galactic rotation ($v_r \sim 250$ km/sec) and the escape speed ($v_{esc} \sim 340$ km/sec) at the Sun's location (~ 8 kpc from the centre of the galaxy). It is clear then that the HISM does not have to partake in Keplerian motion around the galaxy as do the stars and cold component of the interstellar medium. Furthermore, unless held back in some manner, it will flow away from the disc of the galaxy forming a galactic wind (7,8).

Assuming that the lower energy cosmic rays and plasma move together, an upper limit to the mass flux involved in the galactic wind can be estimated from the total mass of the HISM and the residence time for cosmic rays, namely $2M_{\odot}$/year. This is an acceptable level of mass loss for the galaxy and implies an outflow speed $V_z \sim 15$ km/sec at the outer edge of the cosmic ray containment volume. An indication of the expected nature of the galactic wind is given in Figure 3. Note that the halo is likely to be roughly spherical since the scale height of the medium, including the cosmic rays, is of the order of 10 kpc. The halo, as well as the whole cosmic ray containment volume, should be maintained in a state

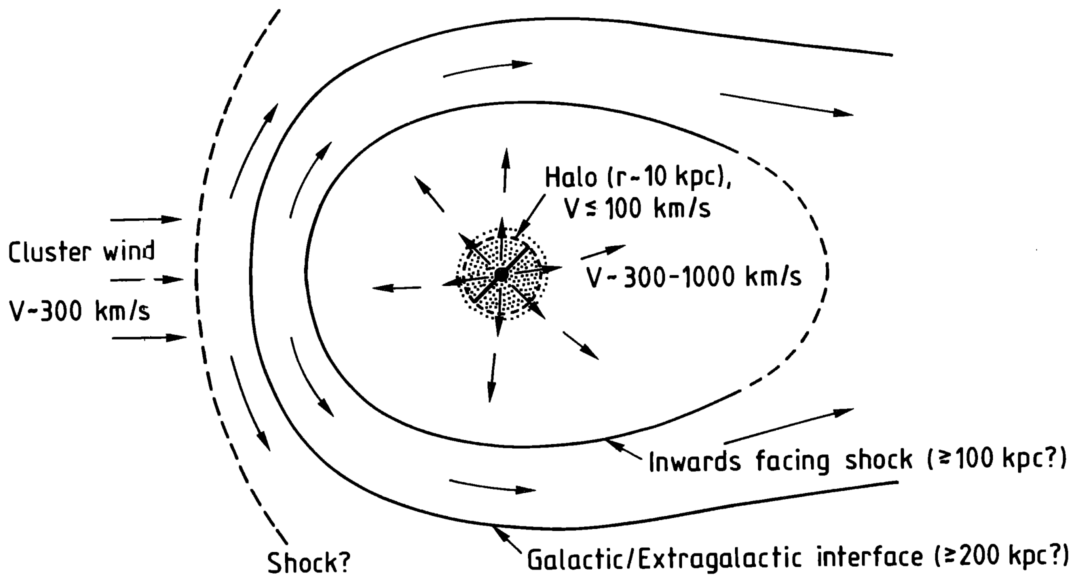


Fig. 3. Possible configuration of the galactic wind and its interaction with the local cluster medium. The numbers shown are mere guesses intended to give some idea of what might be expected. Since the plasma density decreases rapidly away from the galaxy (first exponentially and further out as the inverse square of the distance), supernova shock waves should become stronger as they propagate through the halo and out into the galactic wind region. Particle acceleration can occur at such shock waves and also at the terminal shock. Extragalactic cosmic rays should be modulated at low energies in a manner similar to the solar modulation of galactic cosmic rays.

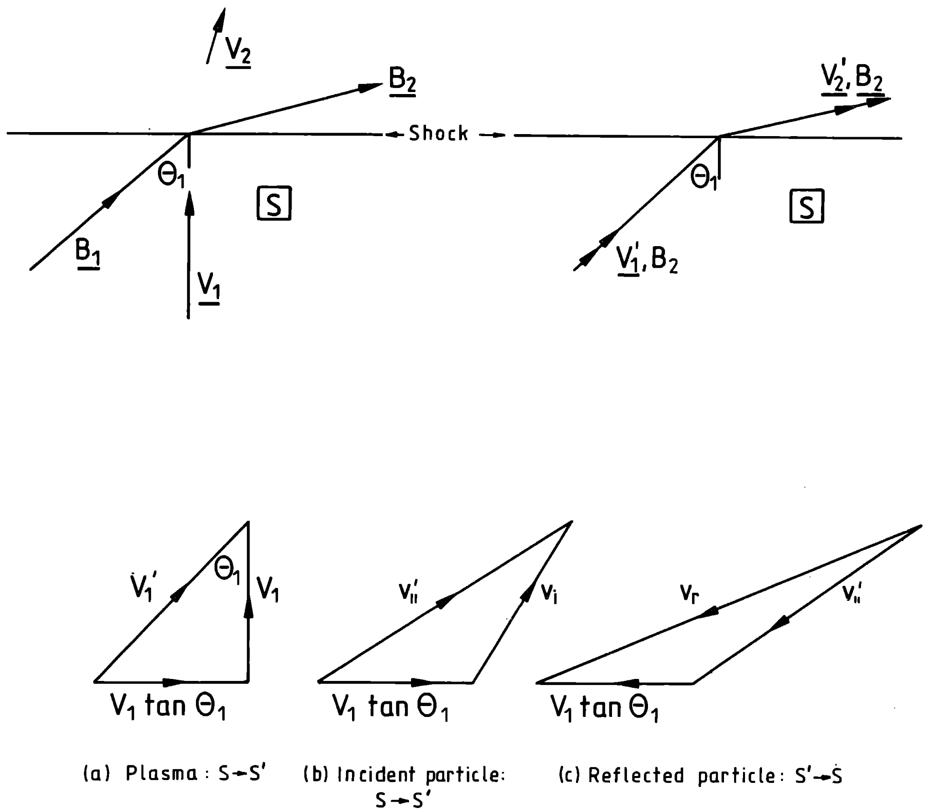


Fig. 4. Above: The flow configuration in the shock frame S (the upstream plasma flow is normal to the shock plane) and in the frame S' in which the electric field vanishes (the plasma flow is parallel to the magnetic field everywhere).
Below: (a) the transformation of the plasma flow S → S';
(b) the transformation of the velocity of an incident particle with velocity v_i in S, to $v_{\parallel}'$ in S';
(c) the transformation of the velocity of a reflected particle, $v_{\parallel}'$ in S' to v_r in S. Note that $v_r \gg v_i$.

of strong supersonic turbulence by interacting supernova shock waves. In particular, shock waves which run out into the halo should tend to speed up and strengthen so that the gas must be continually heated in a manner similar to that envisaged in early theories for the heating of the solar corona (121).

If it is assumed that the escape of cosmic rays from the containment volume is diffusive it is possible to estimate a value for the diffusion coefficient normal to the plane of the galaxy as:

$$\kappa_z = L^2/3\tau \sim 2\mathrm{x}10^{26}(T/T_1)^{0.35}\mathrm{cm}^2/\mathrm{sec}\ , \qquad (3.2)$$

where $L \sim 200$ parsecs is the distance to the escape boundary. The corresponding mean free path is $\lambda_z \sim 0.06(T/T_1)^{0.35}$pc, which is larger than the gyroradius $r_g \sim 10^{-6}(T/T_0)$ pc, provided $T \lesssim 10^6$ GeV. Large-scale convection by the galactic wind in the containment volume could have important effects at low energies since $V_zL/\kappa_z \sim 3$ for $T = T_1$, if $V_z =$ 10 km/sec.

4. Scatter-free Shock Acceleration

In a perpendicular shock (i.e. propagating perpendicular to the magnetic field) it has been shown that, if scattering can be neglected, particles interacting with the shock conserve their magnetic moments (122-126,11). Thus,

$$p^2_{\perp 1}/B_1 = p^2_{\perp 2}/B_2 \qquad (4.1)$$

where subscripts 1/2 refer to upstream/downstream conditions respectively and p is the particle momentum. Subsequent expansion of the medium back to the original magnetic field strength will return the particle energy to its original value unless pitch angle scattering occurs during the intermediate phase. We cannot rely only on this mechanism to account for the acceleration of galactic cosmic rays, however it demonstrates that shocks have an important effect on the existing cosmic rays which cannot avoid having their energy density enhanced by ~ 4 and (for electrons) their synchrotron volume emissivity enhanced by a factor ~ 100 for strong shocks.

Oblique shocks are more interesting in that incident particles can be reflected back upstream with a considerable increase of energy in favourable circumstances (43-46,11,14,123,127-132). In order to understand this mechanism it is advisable to make a transformation from the "shock frame" (S) in which the shock is stationary and the incident flow normal to it, to the frame (S') in which the ambient electric field vanishes (Figure 4). The transformation involves a velocity change $(-\ V_1\tan\theta_1,0,0)$ which makes the bulk flow velocity parallel to the magnetic field on both sides of the shock. In the simplest situation where magnetic reflection occurs, the particle energy is conserved in S' and it can be shown that for θ_1 sufficiently large, the particle magnetic moment is also approximately conserved (12,14,130). Using primes to denote quantities in S', the condition for reflection is $T_\perp' > (B_1/B_2)T'$ (i.e. the particle must have a sufficiently large pitch angle in S'). On transforming back to S, it is found that for reflected particles:

$$\Delta T_{ref} = 2mv_\parallel' V_1 \sin\theta_1 \tan\theta_1 \qquad (4.2)$$

and $T_{\perp 2} = (B_2/B_1) T_{\perp 1}$ for non-relativistic transmitted particles. If the incident particle is relativistic, $\Delta T/T \sim 2(V_1/c)\sin\theta_1\tan\theta_1$, which can be large if $\theta_1 \to \pi/2$. For low energy particles $\Delta T/T$ can exceed unity but the effect is not interesting unless the initial particle energy is already of order $(mV_1^2\tan^2\theta_1/2)$. Thus, for example, an incident particle having a guiding centre moving with the upstream medium cannot be reflected unles its initial energy exceeds $1/2\ mV_1^2(B_2+B_1\tan^2\theta_1)/(B_2-B_1)$, which is already large if $\tan^2\theta_1$ is large enough for the energy change on reflection to be substantial ($\Delta T = 2mV_1^2\tan^2\theta_1$).

This mechanism cannot be a basis for the acceleration of galactic cosmic rays (133) since it depends on the prior existence of a "seed" population which is more energetic than the background plasma and therefore subject to severe energy losses between shock encounters which would suppress the heavier particles, contrary to observation (114). Nevertheless, the mechanism exists and must be taken into account, for example in consideration of the shock transition conditions in analyses based on cosmic ray transport equations in scattering media (see section 5). Furthermore, in interplanetary space where suprathermal "seed" particles are quite prevalent, the mechanism can produce the "shock spike" phenomenon as a result of slow variations of the magnetic field direction which temporarily make $\theta_1 \to \pi/2$.

A variant on the mechanism has been proposed by Sonnerup (134) who suggests that reflection processes other than magnetostatic may be effective and that as a consequence a small fraction of the incident thermal plasma might be reflected and accelerated. Without specifying the reflection mechanism, it is easily shown that the energy of reflected ions would be $mV_1^2(1+4\tan^2\theta_1)/2$ if the incident Mach number is large and magnetic moments are conserved. Observations indicate that this process may be operating on solar wind particles at the Earth's bow shock (135) which suggests that it might be able to provide an injection mechanism for galactic cosmic rays in supernova shocks. It is of course necessary to be sure that the effect is different from magnetostatic reflection of suprathermal particles and/or leakage of shock-heated ions upstream, both of which would produce beams with energies $\propto \tan^2\theta_1$ as a result of the velocity "filter" arising from the interaction of the solar wind with a finite shock.

It is interesting to speculate on possible reflection mechanisms other than the simple magnetotstatic one usually assumed. One possibility is an electric field normal to the shock surface which tends to reflect ions and holds electrons behind the shock. A simple analysis shows that for a cold (high Mach number) incident plasma, such an electric field would have to have a potential drop of at least $mV_1^2/2e$ ($\sim$ 1 kV in the solar wind) to have a significant effect, whereas one would expect potential drops of about kT_e/e ($\sim$ 30-50 volts) if the shock is not perpendicular, so that the effect would be of secondary importance at best.

A second possibility is reflection from the magnetic field "overshoot" that occurs at the front of quasi-perpendicular shocks, where B_2 may be locally a factor $\sim$ 2 greater than predicted by the Rankine-Hugoniot conditions (136); however, for a high Mach number incident plasma the magnetostatic reflection requirement is difficult to satisfy so that even with the assistance of an electric potential drop it seems unlikely that this could explain the observations. A third possibility is reflection by strong turbulence which could scatter the particles back upstream. This cannot be Alfvén wave turbulence, however, since in the frame S', Alfvén waves are convected away from the shock at a speed exceeding $V_1 \tan\theta_1$ and for $\tan\theta_1 >> 1$ the reflection would involve an energy loss which is barely compensated in transforming back to the shock frame. To be effective, the waves must be stationary or even move towards the shock in S'; large amplitude whistlers, which are prevalent in oblique shocks, could satisfy this requirement although it is not immediately clear whether they could efficiently scatter ions.

5. Acceleration by Shocks in Scattering Media

In contrast to the scatter-free mechanism described in section 4, in which particles make a single interaction with the shock before being reflected or transmitted downstream, the presence of scattering permits multiple interactions to occur and hence leads to much more efficient acceleration. It is possible (4,137-140) to deduce the simplest result for acceleration by a plane steady shock by random walk arguments as discussed by Chandrasekhar (141), but in general it is best to use transport equations which describe the behaviour of the particle distribution in terms of convection and diffusion in the scattering medium, including energy changes due to various causes. In terms of the differential current S(x,T)dT and number density U(x,T)dT in (T, T+dT), the one-dimensional forms of these equations are (56):

$$\frac{\partial U}{\partial t} + \frac{\partial S}{\partial x} = -\frac{1}{3} V \frac{\partial^2}{\partial x \partial T} (\alpha T U) + Q , \qquad (5.1)$$

$$S = CVU - \kappa \frac{\partial U}{\partial x} = V(U - \frac{1}{3} \frac{\partial}{\partial T} (\alpha T U)) - \kappa \frac{\partial U}{\partial x} , \qquad (5.2)$$

which can be combined to yield a Fokker-Planck equation for U (52-56):

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x} (VU) = \frac{\partial}{\partial x} (\kappa \frac{\partial U}{\partial x}) + \frac{1}{3} \frac{\partial V}{\partial x} \frac{\partial}{\partial T} (\alpha T U) + Q . \qquad (5.3)$$

Here V(x) is the speed of the scattering medium, Q represents particle sources and sinks and non-adiabatic energy changes, $\kappa(x,T)$ is a diffusion coefficient related to the spectrum of magnetic irregularities in the medium (142) and C is the Compton-Getting coefficient (143,144). In deriving these equations, which are correct to a factor $(1 + O(V/v)^2)$, it is assumed that the distribution function is nearly isotropic (S <<

$vU/3$), the Compton-Getting transformation is linear and particle inertia effects are negligible ($\partial S/\partial t \ll v^2 S/3\kappa$). In terms of U, S the differential intensity is $j = vU/4\pi$ and the anisotropy is $\xi = 3S/vU$.

Consider the situation in which a shock wave is situated at $x = 0$ facing in the negative x-direction so that $V = V_1$ in $x < 0$, $V = V_2$ in $x > 0$ with $V_1 > V_2$. We assume that $U \to U_1(T)$ as $x \to -\infty$, U remains finite as $x \to \infty$ ($U \to U_2(T)$) and there is a source $Q = Q_o(T)\delta(x)$ corresponding to particle injection at the shock. It is easily shown that if the above transport equations remain valid throughout, the transition conditions appropriate to a sudden change in $V(x)$, $\kappa(x,T)$ etc., are:

$$U_1 = U_2 \ , \quad S_1 + Q_o = S_2 \ , \tag{5.4}$$

where the subscripts refer to conditions upstream and downstream of the transition respectively (56,11). [If the diffusion coefficient is anisotropic and the shock oblique, the condition on the current is $S_{n1} + Q_o = S_{n2}$.] In fact, the transport equations are not valid on such a small scale (transition thickness << mean free path) but a more detailed treatment shows that the conditions (5.4) are correct provided the distribution function does not become very anisotropic at the shock (11,145). For low energy particles, shock reflection as described in section 4 may introduce a correction $(1 + O(V/v))$ to these conditions which would not invalidate the use of the transport equations elsewhere (145).

Provided $\int_o^x dx'/\kappa(x',T) \to 0$ as $x \to -\infty$, the solution of equations (5.1), (5.2) in this situation is

$$\begin{aligned} U(x,T) &= U_1(T) + \{U_2(T) - U_1(T)\} \exp \int_o^x \{V_1/\kappa(x',T)\}dx' \ , \quad x < 0 \ , \\ &= U_2(T) \ , \quad x > 0 \ , \end{aligned} \tag{5.5}$$

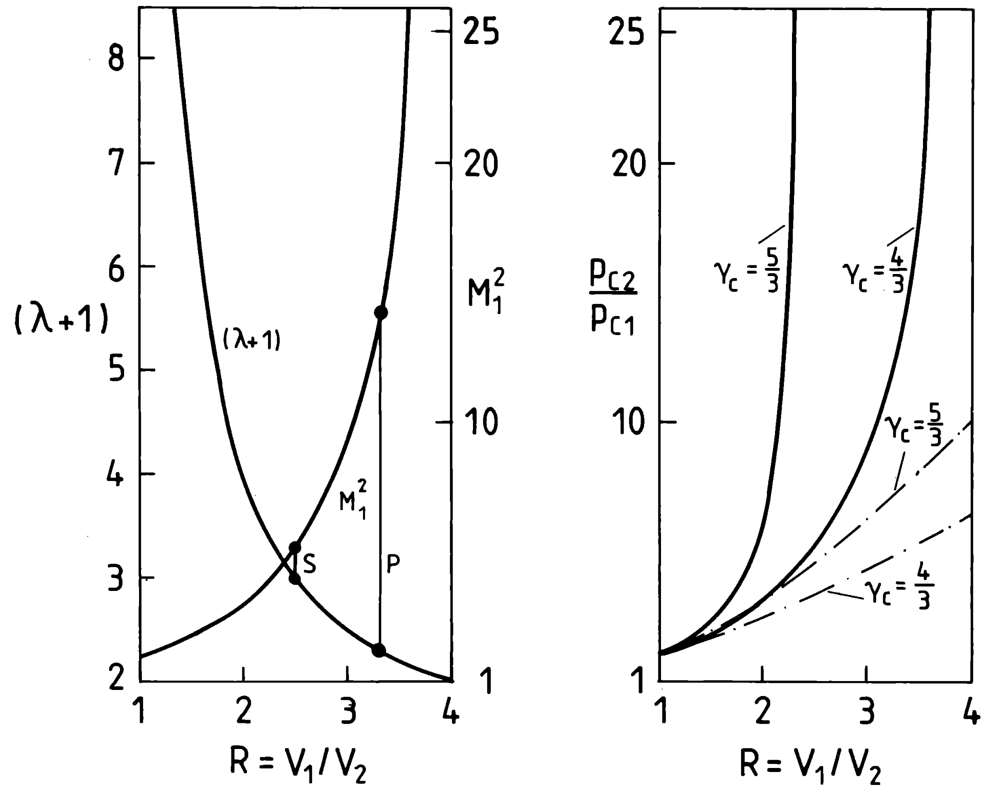
where $U_2(T)$ is determined from the condition on $S(T)$ at $x = 0$:

$$\frac{1}{3}(V_1 - V_2)\frac{\partial}{\partial T}(\alpha T U_2) + V_2 U_2 = V_1 U_1 + Q_o \ . \tag{5.6}$$

This equation can be integrated to yield

$$\alpha T U(T)\{T(T+2T_o)\}^{\lambda_o/2} = (V_1\lambda_o/V_2)\int_o^T \{T'(T'+2T_o)\}^{\lambda_o/2}\{U_1(T') + Q_o(T')/V_1\}dT' \ , \tag{5.7}$$

where $\lambda_o = 3V_2/(V_1 - V_2)$ (2-5). In particular if either $U_1(T)$ or $Q_o(T)/V_1$ have the form $\alpha_1 T_1 U_1 \delta(T - T_1)$, then



(a) (b)

Fig. 5. (a) Spectral index $(\lambda+1)$ as a function of compression ratio $(R = V_1/V_2)$ and upstream Mach number (M_1) for a plane steady shock with monoenergetic injection (equation 5.9). The shocks responsible for the primary production spectrum correspond to the points labelled p and those which first affect the secondary spectrum to the points labelled s.
(b) Cosmic ray pressure ratio across a plane, steady shock (p_{c2}/p_{c1}) as a function of compression ratio for γ_c = 5/3, 4/3 from equation (5.11) (solid lines). The pressure ratio obtained from adiabatic compression with the same compression ratio is substantially less (dashed lines) except for weak shocks.

$$\alpha TU_2(T) = (V_1\lambda_o/V_2)\alpha_1 T_1 U_1\{T_1(T_1+2T_o)/T(T+2T_o)\}^{\lambda_o/2}\ , \ T \geq T_1\ . \quad (5.8)$$

Alternatively, if we treat $\alpha = \bar{\alpha}$ as constant, the equivalent solution is

$$U_2(T) = (V_1\lambda/V_2)\ U_1(T/T_1)^{-(\lambda+1)}\ ,\quad T \geq T_1\ , \quad (5.9)$$

where $\lambda = \lambda_o/\bar{\alpha}$. Thus the shock converts a monoenergetic distribution into a power law distribution above the initial energy, with a spectral index depending on the compression ratio $R = V_1/V_2 = 4M_1^2/(3+M_1^2)$ where M_1 is the shock Mach number ($\gamma = 5/3$) (see Figure 5a). For strong shocks, $M_1^2 \to \infty$, $R \to 4$ and hence $\lambda_o \to 1$ and $(\lambda+1) \to 3/2$, for non-relativistic and $\to 2$ for relativistic particles, respectively. The pre-shock density increase is exponential (if κ remains finite as $x \to -\infty$) and the spectrum in this region depends on the form of the diffusion coefficient (Figure 6a). Note that the power law spectrum is achieved only under the conditions we have assumed; if a 'free-escape' boundary ($U = 0$) is assumed at $x = -L$ for example, exponential spectra are obtained (146).

Equations (5.7) and (5.8) have a neater form if expressed in terms of the particle momentum (p) and the omni-directional component of the distribution function $F_o(p) = \alpha TU(T)/4\pi p^3$:

$$F_{o2}(p) = (3/(V_1-V_2))Q_o(p/p_1)^{-\lambda_1}\ , \quad (5.10)$$

where $\lambda_1 = (\lambda_o+3) = 3R/(R-1) \to 4$ for strong shocks.

It is instructive to consider the behaviour of the cosmic ray pressure according to these results. From (5.9) we obtain

$$p_{c2}/p_{c1} = V_1\lambda/V_2(\lambda-1) = R/\{1 - \bar{\alpha}\ (R-1)/3\} \quad (5.11)$$

and find that $p_{c2}/p_{c1} \to \infty$ for $R = 1+3/\bar{\alpha}$ (i.e. $R = 2.5$ for non-relativistic and $R = 4$ for relativistic particles). The divergence in the downstream pressure is a result of the flatness of the spectrum and indicates that shocks tend to bring particles to high energies. Furthermore, on comparing the cosmic ray pressure jump to that which would result from an equivalent adiabatic compression, one sees that shocks give more energy to the cosmic rays than an adiabatic change and the process is therefore irreversible, as would be expected from (5.9), (5.10) (see Figure 5b).

The effect of shock acceleration in the case in which the upstream cosmic ray spectrum is a power law is of particular interest in considering the post-acceleration of secondary particles. Using (5.7) with $Q_o = 0$, $U_1(T) = U_1(T/T_1)^{-\mu}$ in $T \geq T_1$ and assuming $T \gg T_o$, one finds that if $(\lambda+1) \neq \mu$

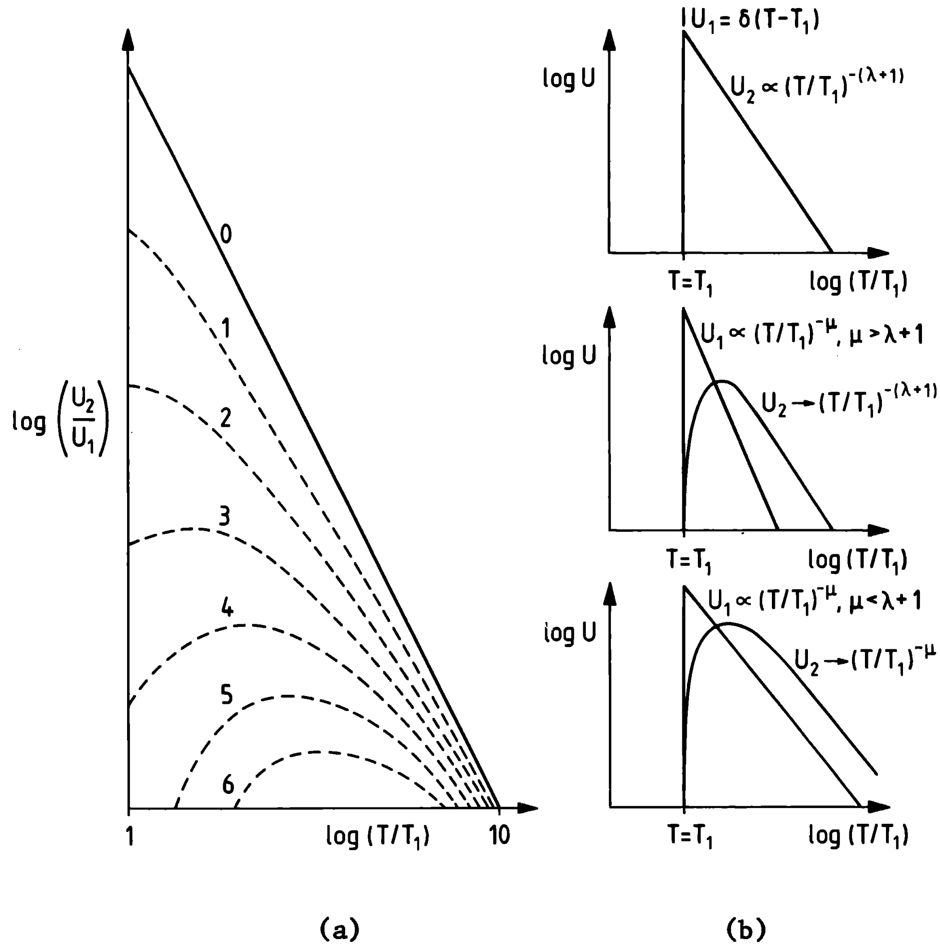


Fig. 6. (a) Spectra of particles accelerated by a strong ($V_1/V_2 = 4$) plane, steady shock with monoenergetic injection at $T = T_1$ (equations (5.5), (5.9)), at various distances (V_1x/κ_1) upstream and with $\kappa = \kappa_1(T/T_1)$. Note that the spectrum is a power law only at the shock and downstream; ahead of the shock it tends to be peaked because high energy particles can diffuse further upstream than low energy particles.
(b) The effect of a shock wave on an incident or injected spectrum which has a power law form above a certain energy. If the initial spectrum is softer than the shock would produce for monoenergetic injection (5.9, 5.10) the downstream spectrum is altered to the "shock" spectrum at high energies. On the other hand, if the initial spectrum is flatter than the "shock" spectrum the initial power law is preserved at high energies but the spectrum is shifted upwards.

$$U_2(T) = \beta U_1 \{(T/T_1)^{-\mu} - (T/T_1)^{-(\lambda+1)}\} , \quad T > T_1 , \qquad (5.12)$$

where $\beta = 1/(1-C_1(V_1-V_2)/V_1) = \lambda V_1/V_2(\lambda+1-\mu)$ and $C_1 = 1+\bar{\alpha}(\mu-1)/3$ is the upstream Compton-Getting coefficient. For $T >> T_1$ we see that $U_2(T) \propto (T/T_1)^{-\mu}$ if $\mu < (\lambda+1)$ and $U_2(T) \propto (T/T_1)^{-(\lambda+1)}$ if $\mu > (\lambda+1)$ as shown in Figure 6b. Thus shock waves which are capable of generating the observed cosmic ray spectrum at high energies (i.e. $\lambda+1 = 2.3$) will change the spectrum of secondaries, but shocks with $(\lambda+1) \geq \mu_2 \sim 3$ will not; the corresponding compression ratios and shock Mach numbers are $R = 3.3$, $M_1^2 = 14.3$ and $R \leq 2.5$, $M_1^2 \leq 5$, respectively.

The solution given here for a plane, steady shock can be generalized to the case in which the diffusion coefficient is anisotropic. If the y-axis is taken to lie in the plane of the shock and the magnetic field to lie in the (x,z) plane, and if $\underline{V} = (V_1,0,0)$ in $x < 0$, then we must put $\kappa_1 = \kappa_{\parallel}\cos^2\theta_1 + \kappa_{\perp}\sin^2\theta_1$. The components of the current vector parallel to the shock surface are:

$$S_y = - \kappa_T \sin\theta_1 \frac{\partial U}{\partial x} , \quad S_z = -(\kappa_{\perp} - \kappa_{\parallel})\sin\theta_1\cos\theta_1 \frac{\partial U}{\partial x} , \qquad (5.13)$$

where $\kappa_T = pv/3eB$ and $\kappa_{\parallel}$, $\kappa_{\perp}$ are the components of the diffusion coefficient parallel and perpendicular to the magnetic field direction, respectively. In the limit $\kappa_{\perp} \to 0$,

$$\frac{S_x - CV_1U}{S_z} = \frac{\kappa_{\parallel}\sin^2\theta_1 + \kappa_{\perp}\cos^2\theta_1}{(\kappa_{\parallel}-\kappa_{\perp})\sin\theta_1\cos\theta_1} \to \tan\theta_1 , \qquad (5.14)$$

thus for $\theta_1 \to \pi/2$ the particle distribution upstream of the shock is highly anisotropic and field-aligned much as in the case of scatter-free acceleration.

6. Plane Shocks With Energy Losses and Time Dependence

Solutions of the plane, one-dimensional problem discussed in section 5 allowing for time dependence with various initial conditions have been given by Fisk (147), Vasil'yev et al. (148), Toptygin (11) and Forman and Morfill (149). The effects of energy losses due to collisions, synchrotron emission and Compton scattering and adiabatic expansion, all expressed in terms of a loss time $\tau(T)$, have been considered by Bulanov and Dogiel (150,151) and Völk et al. (152-154).

If particles are injected at the shock, beginning at $t = 0$, with a monoenergetic spectrum (i.e. $U_1(T) = 0$, $Q_o(T,t) = Q_o\delta(T-T_1)H(t)$) and there are distributed losses of the form $Q(T) = -U/\tau$, equations (5.1) and (5.2) can be written

$$\frac{\partial \bar{S}}{\partial x} = - \frac{1}{3} V \frac{\partial^2}{\partial x \partial T} (\alpha T \bar{U}) - \bar{U}(s + \frac{1}{\tau}) , \qquad (6.1)$$

$$\bar{S} = V(\bar{U} - \frac{1}{3}\frac{\partial}{\partial T}(\alpha T\bar{U})) - \kappa \frac{\partial \bar{U}}{\partial x} \quad , \tag{6.2}$$

where bars represent a Laplace transformation with respect to time (e^{-st}). Assuming κ, V to be independent of x both upstream and downstream of the shock, solutions of these equations exist such that $\bar{U}$, $\bar{S}$ $\propto \exp \beta(s)x$, where

$$\beta = \beta_1(s) = [1 + \surd\{1 + 4\gamma_1\kappa_1/V_1^2\}](V_1/2\kappa_1) \quad \text{in } x < 0 \ , \tag{6.3a}$$

$$\beta = \beta_2(s) = [1 - \surd\{1 + 4\gamma_2\kappa_2/V_2^2\}](V_2/2\kappa_2) \quad \text{in } x > 0 \ , \tag{6.3b}$$

and $\gamma_i = s + 1/\tau_i$. Applying the continuity conditions (5.4) in the form

$$\bar{U}_1 = \bar{U}_2 = \bar{U}_o(T) \ , \quad \bar{S}_1 + Q_o\delta(T-T_1)/s = \bar{S}_2 \ , \quad \text{at } x = 0 \ , \tag{6.4}$$

we eventually obtain for $\bar{U}_o(T)$ (the density of particles at the shock):

$$\bar{U}_o(s,T)/U_2(T) = \frac{1}{s} \exp \int_{T_1}^{T} - \frac{3}{2}\frac{(V_1A_1+V_2A_2)}{(V_1-V_2)}\frac{dT'}{\alpha T'} \ , \tag{6.5}$$

where $U_2(T)$ is the steady state solution given by (5.8) with $Q_o = \alpha_1 T_1 U_1$ and $A_i = \surd\{1+4\gamma_i\kappa_i/V_i^2\}-1$. The effects of time dependence and energy losses are contained entirely in the exponential term on the right of (6.5) and they are not important if A_1, $A_2 \to 0$ (i.e. $4\gamma_i\kappa_i/V_i^2 \ll 1$).

Consider first the case of time dependence without energy losses ($\tau_i \to \infty$). Although the inversion of (6.5) is difficult in general, simple approximations can be found for large and small values of the time and an exact solution is available in a special case in which $A_1 = A_2$ and κ_1, κ_2 do not depend on energy. For small s we can take $V_iA_i \sim 2s\kappa_i/V_i$ and hence obtain a rough representation of the solution for large time:

$$U_o(T,t) \sim U_2(T)\ H(t-\tau_a) \ , \tag{6.6}$$

where

$$\tau_a = \frac{3}{(V_1-V_2)}\int_{T_1}^{T}\left[\frac{\kappa_1}{V_1} + \frac{\kappa_2}{V_2}\right]\frac{dT'}{\alpha T'} \ . \tag{6.7}$$

That is, at a given time t, no particles are seen at energies greater than a value T such that $\tau_a(T) = t$, while at lower energies the spectrum approximates that of the steady state solution (5.8).

For the case in which $4\kappa_1/V_1^2 = 4\kappa_2/V_2^2 = t_o$ (independent of T) equation (6.5) can be inverted to yield

$$U_o(T,t)/U_2(T) = \{(T+2T_o)T/(T_1+2T_o)T_1\}^{\nu}\log\{(T_o+2T_o)T/(T_1+2T_o)T_1\}^{\nu}$$

$$\times \int_o^{t/t_o} \exp\left[-\frac{\nu^2}{4t'}\left(\log\{(T+2T_o)T/(T_1+2T_o)T_1\}\right)^2 - t'\right] \frac{dt'}{\sqrt{(4\pi t'^3)}}\ , \quad (6.8)$$

where $\nu = 3(V_1+V_2)/4(V_1-V_2)$. Alternatively, in terms of particle momentum:

$$F_o(p,t,x=0)/F_{o2}(p) =$$

$$2\nu(p/p_1)^{2\nu}\log(p/p_1)\int_o^{t/t_o} \exp\left[-\frac{\nu^2}{t'}(\log p/p_1)^2 - t'\right]\frac{dt'}{\sqrt{(4\pi t'^3)}}\ , \quad (6.9)$$

where $F_{o2}(p)$ is the steady state solution given by (5.10). Using Laplace's method to estimate the integrals in these expressions for sufficiently large t/t_o we find that they behave like (somewhat spread out) step functions with the "step" occurring at $t = \nu t_o \log(p/p_1) = \tau_a$ as defined in (6.7) (see Figure 7). For $t >> \tau_a$ one finds that the right hand sides of (6.8) and (6.9) tend to unity as expected. [Note that these results are slightly different from those given by Toptygin (11) who effectively takes $Q_o(T,t) = Q_o\delta(T-T_1)\delta(t)$.]

The effects of energy losses on the steady state spectrum can be found by taking the limit $s \to 0$ in (6.5) and inverting:

$$U_o(T)/U_2(T) = \exp\int_{T_1}^{T} -\frac{3}{2}\frac{(V_1A_1+V_2A_2)}{(V_1-V_2)}\frac{dT'}{\alpha T'}\ , \quad (6.10)$$

where $A_i = \sqrt{\{1+4\kappa_i/V_i^2\tau_i\}}-1$. In the case in which A_1 and A_2 are independent of energy this integral is easily evaluated:

$$U_o(T)/U_2(T) = \left[(T+2T_o)T/(T_1+2T_o)T_1\right]^{-\eta}\ , \quad (6.11a)$$

or alternatively

$$F_o(p,x=0)/F_{o2}(p) = (p/p_1)^{-2\eta}\ , \quad (6.11b)$$

where $\eta = 3(V_1A_1+V_2A_2)/4(V_1-V_2)$. The spectrum with losses is considerably steeper than without if $\eta = O(1)$. In general, we require that $\tau_1,\tau_2 >> \tau_a$ if losses are not to effectively quench the shock acceleration mechanism. An example showing the spatial variations of various values of $A_1 = A_2 = \sqrt{(1+X)}-1$ is shown in Figure 8.

It should be noted that the "loss time" approximation to the energy loss term in (6.1) is not strictly valid except in the case of catastrophic losses due to nuclear interactions, where $1/\tau = vn\sigma$ and $\sigma \sim \pi(1.26\text{x}10^{-13} A^{1/3})^2$ cm^2, with n being the number density in the background

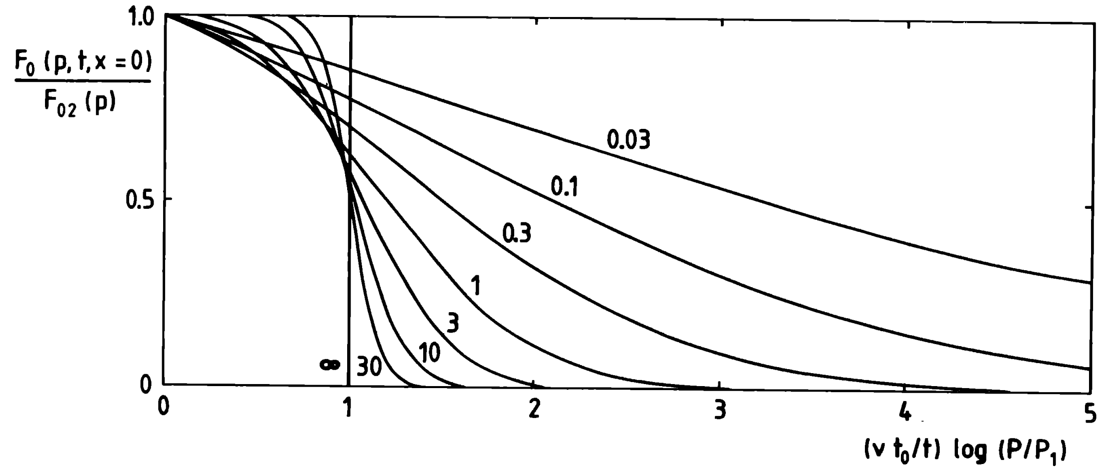


Fig. 7. The time dependent development of the spectrum at the shock for energy independent diffusion coefficients and monoenergetic injection (equation 6.9). We show here the ratio of the spectrum at a given time t to the equilibrium spectrum as a function of momentum.

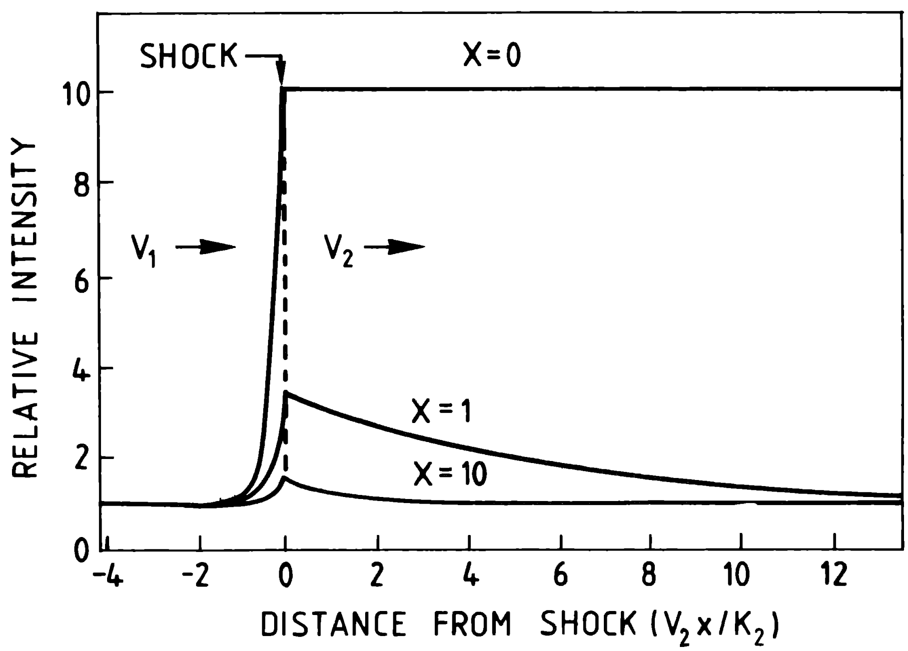


Fig. 8. The effect of losses ($X = 4\kappa_2/V_2^2\tau_2$) on the efficiency of shock acceleration (Völk et al. (152-154)).

medium and A the atomic number of the energetic particles. For continuous energy losses, the approximation is relatively crude, since in fact:

$$Q = -\frac{\partial}{\partial T}\left[U \sum_i \left(\frac{dT}{dt}\right)_i\right] , \qquad (6.12)$$

where, in units of MeV/sec and with T expressed in MeV/nuc

$$\frac{dT}{dt} = 2.5\text{x}10^{-9}\ B^2T^2 \quad \text{for synchrotron losses } (e^+,e^-) ,$$

$$= 8\text{x}10^{-16}\ nT \quad \text{for Coulomb collisions} ,$$

$$= 4.8\text{x}10^{-8}\ W_rT^2 \quad \text{for inverse Compton losses } (e^+,e^-),$$

$$= 4.5\text{x}10^{-13}\ nZ^2(c/v) \quad \text{for ionization losses} ,$$

$$= (2V/3R_s)\alpha T \quad \text{for adiabatic losses} ,$$

with W_r the energy density of photons, Z the charge state of the particle and div $\underline{V} \sim 2V/R_s$ (155). In these cases, it is possible to use the "loss time" approximation as above, but it is obviously better to make a more exact analysis.

It is possible in principle to solve problems involving continuous energy losses analytically; such solutions have been given for cases in which the losses are adiabatic (156) and due to synchrotron and/or inverse Compton emission (157).

A somewhat related problem concerns the effect of second order Fermi acceleration occurring behind the shock (59,158-160,161). Here, following Webb (161), we take

$$Q = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial F_o}{\partial p}\right) , \qquad (6.13)$$

where $D_{pp} = D_2p^2\exp(-x/L_2)$ in $x > 0$. For the case of monoenergetic ($p = p_1$) injection at the shock and with κ_1, κ_2 independent of p, the solution for large p/p_1 far downstream has the form:

$$F_o(x,p) \propto (p/p_1)^{-\mu} , \qquad (6.14)$$

with

$$\mu = \frac{1}{2}\left[\{3-\eta(1+\eta)(R-1)/3\xi\}^2 + 4\eta(1+\eta)R/\xi\right]^{1/2}$$

$$+ \frac{1}{2}\{3-\eta(1+\eta)(R-1)/3\xi\} , \qquad (6.15)$$

where $\eta = V_2L_2/\kappa_2$ and $\xi = D_2L_2^2/\kappa_2$. For small ξ it can be shown that

$$\mu \rightarrow \frac{3R}{R-1}\left(1 - \frac{9}{2}\xi/\eta(1+\eta)(R-1)^2 + \ldots\right) , \qquad (6.16)$$

and hence the spectrum is flatter at high energies than found in the case without second order Fermi acceleration ($\xi = 0$). In a more general situation where κ_2 and D_{pp} depend on the charge/mass ratio of the particle species one would expect that the acceleration is selective.

7. Acceleration by Non-Planar Shocks

With one exception, namely the model of an interplanetary corotating interaction region given by Fisk and Lee (162), all attempts to deal with the acceleration of energetic particles by non-planar shocks have been restricted to cases of spherical symmetry. The essential difficulty is that the flow on either side of the shock is divergent (div $\underline{V} \neq 0$, in contrast to the one-dimensional case), so that energy changes occur due to adiabatic expansion and compression in addition to the acceleration associated with the shock. Furthermore, in a non-planar geometry, energetic particles may diffuse towards the shock from downstream and also escape upstream relatively easily, whereas in plane shocks convection usually dominates the behaviour at large distances, both upstream and downstream. It is not surprising then, that the energy/momentum spectra produced by non-planar shocks are not in general power laws and that the acceleration produced by the shocks should in many cases be partly undone by adiabatic expansion in the downstream region.

Approximate solutions to non-planar problems can be obtained if one is willing to assume that as far as the upstream medium is concerned, the shock can be treated as being locally plane (i.e. $\kappa_1/V_s \ll R_s$, the shock radius) and the downstream region has $\kappa_2 = 0$, so that convection and adiabatic energy changes dominate (163), or the downstream flow is incompressible so that only convection and diffusion occur (164,165) Several quite complicated problems, in some cases with time dependence, have been treated assuming that the energy/momentum spectrum preserves a particular power law form everywhere (57,58,3) but in view of the results obtained in section 5 this is hardly justifiable (it amounts to neglecting a particular solution). A partial correction of this deficiency can be attempted by combining two power laws above a certain energy and always ensuring that the spectrum is smooth (as shown in Figure 6b for example) (166); however, this procedure is not strictly correct and leads in general to non-conservation of particle numbers.

Two spherically symmetric problems have been solved exactly, namely the cases of a stellar wind with a stationary inwards-facing terminal shock surrounded by an extended region of essentially incompressible

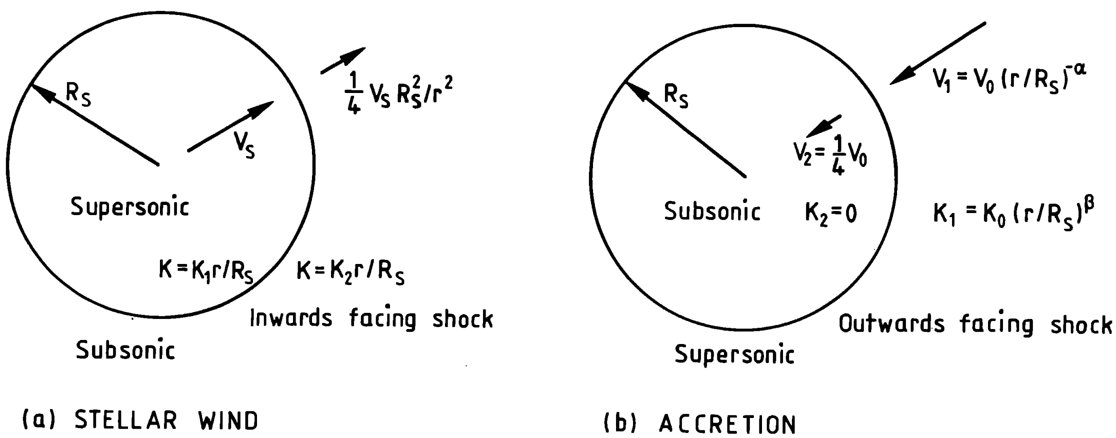


Fig. 9. Configurations assumed for solutions of problems corresponding to shock acceleration at (a) a stellar wind terminal shock (167-169), (b) an accretion shock (170).

flow extending to infinity (167-169) and a stellar accretion flow with a stationary outwards-facing shock, within which convection is dominant (170). The configurations are shown schematically in Figure 9. In both cases the energy/momentum spectrum has a given form and the radial current vanishes at $r = \infty$ and particles may also be injected at the shock ($r = R_s$).

The solution of the terminal shock problem for which $F_o(r,p) \to \delta(p-p_1)$ as $r \to \infty$ and $r^2 S = 0$ at $r = 0$ has the form

$$F_o(r,p) = g(r,p)\left[1-H(p-p_1)\right] + h(r)(p/p_1)^{-\mu}H(p-p_1) \; , \qquad (7.1)$$

where $g(r,p)$ and $h(r)$ are known functions and μ is a function of V_1R_s/κ_1, V_2R_s/κ_2 and V_2/V_1. The second term on the right of this expression is associated with shock acceleration, being a power law as in the plane case, but with a spectral index no longer dependent only on V_2/V_1. The first term on the right represents the effects of deceleration within the stellar wind region (i.e. modulation) which causes some particles to lose energy but in general also leads to a net acceleration (168). The cosmic ray energy flux $\int 4\pi r^2 STdT$ is positive and finite at infinity.

It has been proposed that stellar winds associated with O-stars could be an important source of galactic cosmic rays since (marginally) enough energy seems to be available (120,171,172). The mechanism has the advantage that it is stationary to a first approximation, so that there is no upper limit to the particle energies that can be achieved due to time dependent effects as described in section 6. However, there is a significant difficulty in that the magnetic field lines in the stellar wind region should be tightly wound Archimedian spirals at the position of the shock. Thus if Ω rad/sec is the rotational speed of the central O-star, the average angle between the field lines and the shock surface is $\phi \sim V_1/\Omega R_s$, which with $V_1 = 2000$ km/sec, $2\pi/\Omega = 10^6$ seconds and R_s = 2 parsecs, yields $\Omega R_s \sim 4\times10^{13}$cm/sec >> c; the shock is therefore almost everywhere perpendicular so that particles appearing at the shock, for example, cannot move upstream and be further accelerated. Furthermore, as a result of the tightly wound up magnetic field, the radial component of the diffusion coefficient in the subsonic region behind the shock is very small and hence the intensity of the galactic cosmic radiation must be substantially depressed, so that few particles can reach the shock from behind.

The accretion problem can be solved exactly provided once again the diffusion coefficient is chosen to be independent of energy/momentum. Taking $V_1 = V_o(r/R_s)^{-\alpha}$, $\kappa_1 = \kappa_o(r/R_s)^{\beta}$ and assuming injection at the shock such that $Q \propto \delta(r-R_s)\delta(p-p_1)$, solutions have been obtained for the case $\alpha + \beta = 1$ with asymptotic forms at high energies such that:

$$F_o(p,r) \sim A_1(r/R_s)^{-\eta\delta}(p/p_1)^{-\delta\lambda_1} \; , \quad \eta > \eta_c \; , \qquad (7.2)$$

$$F_o(p,r) \sim \{B_1-B_2 \log(r/R_s)\}(r/R_s)^{-m}\log(p/p_1)(p/p_1)^{-n}, \; \eta<\eta_c, \qquad (7.3)$$

where $\eta = V_o R_s/\kappa_o$, $\eta_c = (1+\beta)(V_o+V_2)/(V_o-V_2)$, $\delta = 1-(1+\beta)V_2/(V_o-V_2)\eta$, $m = 1/2(1+\beta+\eta)$, $n = 3(1+\beta+\eta)^2/4\eta(1+\beta)$ and A_1, B_1, B_2 are constants. Although in general, the spectrum is essentially a power law, the spectral index is affected by the diffusion coefficient through η. For $\eta \to \infty$, the spectral index approaches that of the plane case ($\delta \to 1$) as expected, but for finite η the spectrum is harder as a result of the additional acceleration associated with the convergence of the flow ahead of the shock. (In this case, the particle pressure becomes infinite at the shock, requiring a non-linear treatment.) For small η the shock produces less efficient acceleration and the spectrum is very soft, such that $n \to 3(1+\beta)/4\eta$.

It has been possible to find exact analytic solutions for the accretion problem only for the case in which κ_1 is independent of momentum. However, an approximate asymptotic solution valid for large values of p/p_1 can be obtained for the case $\kappa_1 \propto (p/p_1)^\gamma$. The essential feature of this solution is that the spectrum becomes exponential:

$$F_o(p) \propto (p/p_1)^{-3/2} \exp\{-\frac{3}{2}\frac{(1+\eta)}{4\eta\gamma}(p/p_1)^\gamma\} \ . \qquad (7.4)$$

In all cases the accelerated particles are eventually convected through the shock since $r^2 S \to 0$ as $r \to \infty$; thus in contrast to the case of stellar winds, the configuration cannot provide a source of cosmic rays to the external world without some further modification, such as a "free escape" boundary somewhere outside the shock wave position.

It has been suggested (170,173) that accretion shocks around neutron stars could be an important source of galactic cosmic rays, at least for energies up to 10^6GeV/nuc. Since many (10^9-10^{10}) neutron stars are likely to exist in the galaxy and a significant number of these (10^4-10^5) are in cool, dense clouds where the energy released by accretion can reach $\sim 4\times10^{36}$ergs/sec, it appears that enough power is available. It is also possible to generate the observed spectrum, although it is not obvious why any particular spectral index should be chosen. In view of the high background plasma densities involved it is also unclear whether or not the diffusion coefficient is sufficiently small that $4\kappa_o/V_o^2 \ll \tau_o$. In any case, an additional assumption is required to permit particles to leave the system, which must lead to a substantial reduction in efficiency as well as a change in the form of the spectrum. There is also of course the problem of angular momentum in the external medium which tends to further reduce the efficiency by reducing the solid angle in which an accretion flow can occur.

8. Acceleration of Cosmic Rays by Supernova Blast Waves

In this section we review our present understanding of cosmic ray acceleration by shock waves associated with expanding supernova remnants assuming that the cosmic rays play no part in the dynamics of the remnant. No exact formal solutions describing shock acceleration by blast waves exist since unsteady flow is involved and the problem is too difficult. One can obtain similarity solutions corresponding to the Sedov blast wave solutions (175) but this requires in general that κ is a function of time, which may not be a very reasonable assumption. An approximate solution has been given by Krimsky et al. (163) however it is rather oversimplified and the characteristic effects associated with three-dimensional flows described in section 7 are absent; it is assumed that $V_s R_s/\kappa_1 \gg 1$ so that the shock is effectively plane and $\kappa_2 = 0$ so that the cosmic rays behind the shock simply expand adiabatically. Nevertheless, despite the oversimplification we can at least proceed in this manner to consider how supernova remnants accelerate cosmic rays provided the limitations of the assumptions are recognized.

In order to obtain a feeling for the behaviour of supernova remnants in the HISM it is useful to consider the implications of the diagram given in Figure 10. Here we have assumed, following section 3, that one supernova with average energy 10^{51} ergs occurs every 30 years in a volume V_g and that the HISM has density $n = 3\times10^{-3}\text{cm}^{-3}$, temperature $T_g = 7\times10^5$K and magnetic field strength B = 1 μGauss. We neglect the effects associated with radiative cooling and the presence of dense clouds (6,118) and assume simply that a supernova remnant can be represented by the Sedov solution with

$$R_s = 13(E_{51}/n_o)^{2/5}\ t_4^{2/5}\ \text{parsecs}\ , \tag{8.1}$$

where E_{51} is the supernova energy in units of 10^{51}ergs, n_o the number density of the HISM (cm^{-3}) and t_4 the age of the remnant in units of 10^4 years. Assuming that the supernovae are uniformly distributed, it is possible to calculate the average number of shocks ($\dot{N}_s(\geq V_s)$) with speed greater than a given value passing any point per year and the fraction ($F\ (\leq R_s)$) of the cosmic ray containment volume V_g which is covered by shocks with radii less than a given value:

$$\dot{N}_s(\geq V_s) = 4\pi R_s^3 \dot{N}_g/3V_g, \tag{8.2}$$

$$F(\leq R_s) = (4\pi\dot{N}_g/3V_g)\int_0^{R_s} \dot{R}_s^3\ dt\ , \tag{8.3}$$

where $\dot{N}_g$ is the supernova rate in the galaxy. These quantities are shown in Figure 10 with various abscissae determined from (8.1) and the plane shock results (5.9) and (5.11). It is important to note that the Sedov phase begins only after about $t_4 \sim 1$, since prior to this the mass ejected by the supernova is not negligible compared with the mass of the HISM overrun by the shock. The Sedov phase ends when the total enthalpy of the ambient HISM existing prior to the passage of the shock is comparable to E_{51} (i.e. at $R = R_c$) which is roughly the point at which the Mach number of the shock ($M_1^2 = V_s^2/(c_g^2 + c_A^2)$) is unity. As shown in Figure 10, this occurs at a radius $R_c \sim 180$ pc. Since the speed of the shock cannot be less than the speed of sound, the Sedov solutions underestimate N_s and F when $R_s = V_s \to \sqrt{(c_g^2 + c_A^2)} \sim 185$ km/sec and at all later phases. It is interesting to note, however, that with the conditions assumed this occurs roughly when $F \sim 1$ so that the whole of V_g is in effect covered by supernova remnants of moderate strength as required to account for the existence of the HISM (6).

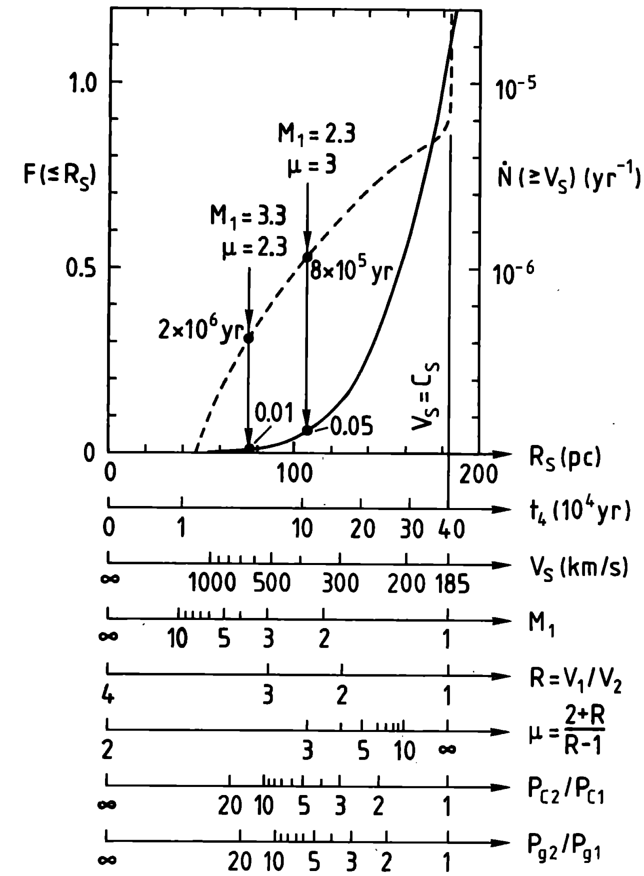


Fig. 10. The rate at which shock waves with speeds $\geq V_s$ pass a given point ($\dot{N}(\geq V_s)$) and the fraction of space in the galaxy covered by shock waves with radii $\leq R_s$ ($F(\leq R_s)$), as functions of shock radius (R_s), age (t_4) and shock speed ($V_s = \dot{R}_s$). We have assumed that the Sedov solution is valid and that the shock waves propagate into the HISM with $n_o = 3\times 10^{-3}$, $E_{51} = 1$, $T_g = 7\times10^5$ K, $B_g = 1$ μGauss and supernova rate $\dot{N}_g = 1/30$ per year. The corresponding values of the Mach number, the cosmic ray spectral index and the cosmic ray and plasma pressure jumps at the shock are also shown. The observed primary cosmic rays must be accelerated predominantly around the region where $\mu = 2.3$, which suggests that the important cosmic ray sources are supernova remnants with radii of the order of 70 parsecs, age 3×10^4 years and occupying about 1% of space. Secondary nuclei produced by interactions taking place throughout the galaxy have their spectral index changed only by shock waves corresponding to $\mu < 3$ and these are seen to occupy only 4% of space.

An immediate implication of Figure 10 is that the shock waves which are largely responsible for determining the cosmic ray source spectrum ($\mu \sim 2.3$) have $M_1 = 3.3$, $R_s \sim 75$ parsecs, $V_s \sim 650$ km/sec, $t \sim 3x10^4$ years and they cover less than $\sim$ 1% of space. The cosmic ray pressure enhancement behind such a shock is of the order of 14 (see Figure 5b); presumably, however, this is something of an overestimate since the plasma pressure enhancement is only of the same order ($p_2/p_1 \sim 5M_1^2/4$ if $M_1^2 >> 1$ and $\gamma = 5/3$) and we have neglected the effects of cosmic ray pressure on the dynamics of the supernova remnant. Evidently, cosmic rays produced in earlier phases (when $\mu < 2.3$) cannot make a significant contribution to the ultimate cosmic ray population due to the overwhelming effects of adiabatic expansion, whereas cosmic rays produced in later phases suffer less from expansion losses but are not accelerated as efficiently. It is important also to note that shocks which alter the observed spectrum of secondaries ($\mu \leq 3$, $M_1 > 2.3$) occupy only $\sim$ 5% of space so that post-acceleration by shocks affects the spectral form of no more than 5% of the secondaries. However, since essentially all of the HISM is covered by shocks (perhaps relatively weak), all secondaries undergo post-acceleration, which must therefore be taken into account in making inferences from their fluxes (as discussed in section 2). The argument (60) that the acceleration of galactic cosmic rays must occur in the HISM because only in this way a large fraction of space can be "processed" is misleading since it implicitly assumes that cosmic rays are accelerated from a seed population rather than emerging from the background plasma; indeed, the above discussion is essentially contradictory to such an approach.

According to Figure 10, the cosmic ray pressure enhancements to be expected are not insignificant in the sense that an enhancement by a factor of 2 or more corresponds to $F \sim 0.35$, $\dot{N}_s \sim 5x10^{-6}$/yr. That is, for 35% of the time and once every $2x10^5$ years, the solar system should be immersed in regions with noticeably enhanced cosmic ray intensity, which is probably not consistent with observations (112,113). This is an indication of the fact that a non-linear approach to supernova dynamics is necessary, leading to a reduction of the cosmic ray pressure enhancement at a given R_s shown in Figure 10, to a reduction of the Mach number (since the effective speed of sound is 200-230 km/sec rather than 185 km/sec) and to corresponding changes in the volumes occupied by the most important cosmic ray producing shocks and the shocks which can affect the secondary spectrum.

Despite the above reservations it is important that the non-self-consistent analysis of cosmic ray acceleration be carried as far as possible to see what further implications it might have. This has been done Blandford and Ostriker (60,61) who made the following assumptions:

(a) the shocks can be considered plane and steady with the accelerated spectrum being determined as described in section 2;
(b) fresh particles are injected at the shock at a constant rate and with a monoenergetic spectrum ($p_1 \sim 100$ MeV/nucleon, say);
(c) the accelerated particles undergo adiabatic expansion by a factor F until the original plasma density or pressure is reached;
(d) specific shock propagation models are used (e.g. Sedov (175) or McKee-Ostriker (6));
(e) the subsequent propagation of cosmic rays takes place as in the simple leaky box model.

On this basis a conservation equation of the following form can be derived:

$$\frac{1}{\tau_s}\left[\int\phi(x-x')Y(x')dx' - Y(x)\right] + Q_o\phi(x) = \text{Losses} + \frac{Y(x)}{\tau_c(x)}\ , \qquad (8.4)$$

where $Y \propto p^4F_o(p)$, $x = \log(p/Z)$, Q_o represents the injection rate, τ_s is a parameter determined by the supernova rate and $\tau_e(x)$ is the residence time of cosmic rays in the galaxy. $\phi(x)$ is the "redistribution function" given by

$$\phi(x) = \int_4^{q'} (q-3)D(q)F(q)^{q-3}\{\exp(4-q)x\}dq\ , \qquad (8.5)$$

where q is the spectral index characterizing the shock of a particular strength, D(q) is obtained from the shock propagation model assumed and q' is determined by the weakest shocks predicted in this model. For reasonable shock propagation models the form of $\phi(x)$ is such that around the injection energy the redistribution corresponds to diffusion in a manner similar to that described by Bykov and Toptygin (11,176,177) for multiple encounters with many weak shocks. In addition, $\phi(x)$ has a high energy tail corresponding to occasional encounters with strong shocks.

The essential result of the work of Blandford and Ostriker is that on the basis of the assumptions listed above, the spectral forms of primary and secondary cosmic rays can be adequately accounted for (see Figure 11). In particular, at high energies (> 10 GeV) the proton energy spectrum has the form $j(T) \propto T^{-(2+\delta+\mu)}$ where in the spectral index, $(2+\delta) \sim 2.2$ results from the initial acceleration by strong shocks and $\mu \sim 0.4$ from the energy dependence of the escape mechanism. The normalization of the spectra and the relative abundances of different primary species are not determined since the injection rate is arbitrary. However, the relative abundances and spectra of secondaries are determined once the Q_o are fixed, just as in the simple leaky box models. The short-comings of this procedure are fairly obvious and are all connected with breakdowns of the validity of the five basic assumptions listed above. In particular, the underlying assumption that cosmic rays can be treated as non-interacting test particles having no effect on the dynamics of the background plasma is not sound as evident from Figure 10.

It is implicit in any model of this class that the energy of any given cosmic ray particle is not constant in time, which has important implications for the interpretation of observations of secondary particles in particular. Most fragmentation products, for example, should be produced in the ISM as a whole, since the product (volume x time x cosmic ray flux) is much greater than in moderately strong supernovae blast waves ($R \lesssim 100$ pc) even taking into account the enhancement of cosmic ray flux in the latter. Nevertheless, the secondaries must be affected by the large number ($\sim 10^2$) of shock waves they encounter during the time they spend in the containment volume. Even if the forms of their spectra are unchanged, the secondaries must be accelerated and

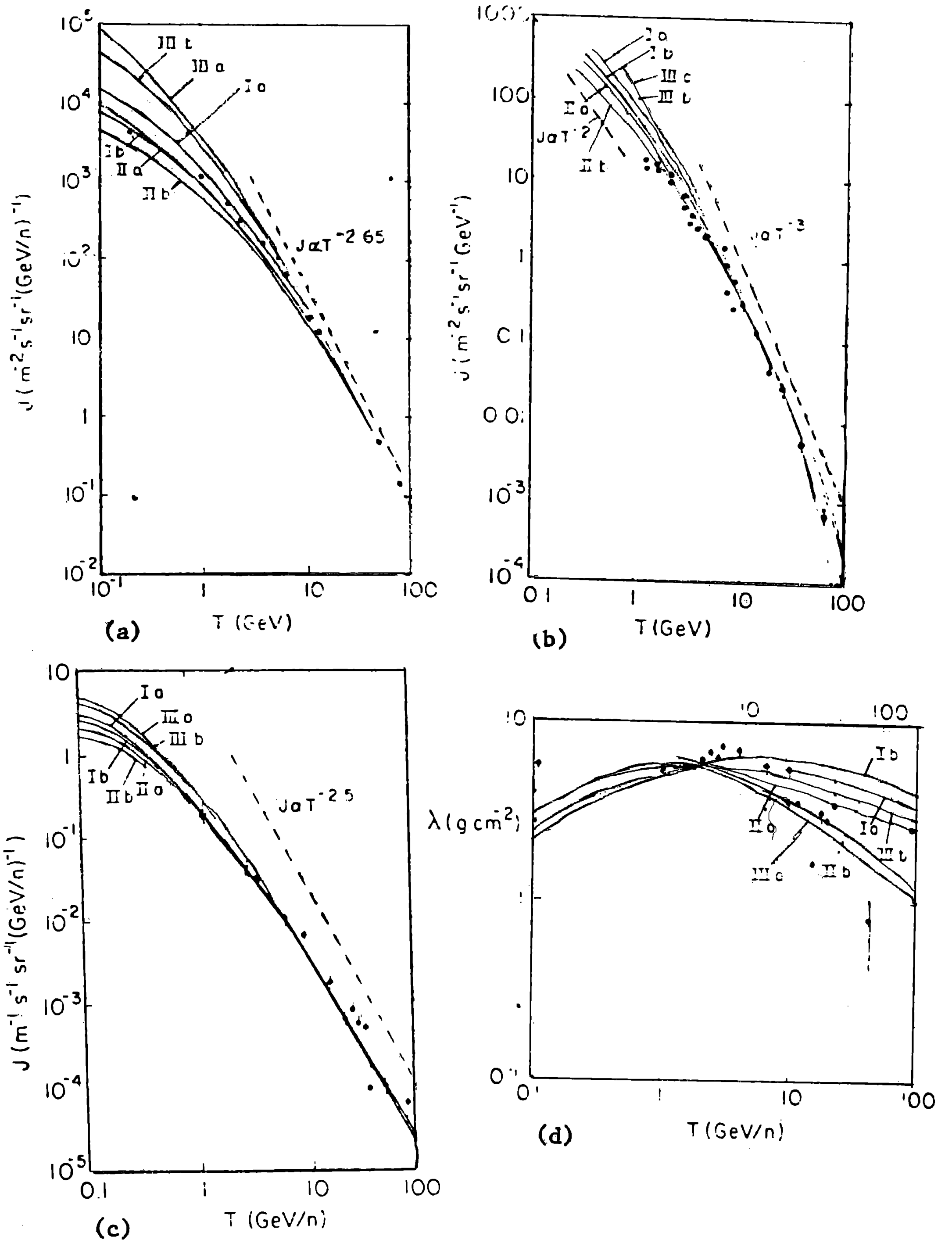


Fig. 11. Model galactic cosmic ray spectra calculated by Blandford and Ostriker (61), for (a) protons, (b) electrons, (c) heavy nuclei and (d) the apparent path lengths of secondaries. Model I corresponds to the supernova model of McKee and Ostriker (6), model II to the Sedov solution and model III is a solution with an assumed power law source and no redistribution. The escape laws used have a simple power law dependence on rigidity (a) and contain an extra rigidity independent term (b).

decelerated (i.e. the spectrum raised and lowered) as a result of such encounters following their formation; if the net energy changes significant, the path length must be correspondingly different from that deduced under conditions of propagation at constant energy. The Blandford-Ostriker model does not provide an especially good fit to the data concerning secondaries (178) but this does not mean that the model is wrong on this account; there are many improvements to be made and it should therefore be regarded as being a first step in the right direction.

Some secondaries, notably anti-protons, are particularly important in this regard, since their production cross sections are strongly energy-dependent (in contrast to fragmentation cross sections). Simple leaky box models with constant particle energy find great difficulty in explaining the presently available measurements of anti-protons, especially at low energies where the direct production rate is very low indeed (179-182). Although a detailed model is not yet available, it seems likely that the anti-protons can only be explained if their production occurs mainly in supernova remnants where the high energy ($T \gtrsim 10^2$ GeV) protons have intensities $\sim 10^4$ greater than normal (partly as a consequence of a flatter spectrum) and is followed by adiabatic expansion bringing significant numbers of anti-protons to energies of $\sim$ 100 MeV. Obviously such secondary particles can provide important information concerning the whole process of cosmic ray acceleration in supernova remnants, which is not obtainable from secondaries produced by fragmentation, for example.

9. The Cosmic Ray Diffusion Coefficient in the HISM

In order for galactic cosmic rays to be accelerated by supernova shocks up to any particular energy it is necessary that the acceleration time $\tau_a \sim 4\kappa_1/V_1^2$ be less than the time available for acceleration $(R_s/\dot{R}_s)$, otherwise the spectrum will exhibit a high energy cut-off. For Sedov blast waves, this condition can be written approximately as $4\kappa_1\dot{R}_s \lesssim R_s/\dot{R}_s$, hence

$$\kappa_1 \lesssim R_s\dot{R}_s/4 = 2\text{x}10^{27}(E_{51}/n_o)^{2/5}\ t_4^{-1/5}\ \text{cm}^2\text{sec}^{-1}\ . \qquad (9.1)$$

With $E_{51} = 1$, $n_o = 3\text{x}10^{-3}$, $t_4 \sim 1$, we require that $\kappa_1 \lesssim 5\text{x}10^{27}$ cm^2/sec. Taking as an estimate $\kappa \sim cr_g/3(\delta B_w/B)^2$ with $r_g \sim 10^{-6}$ T(GeV)pc and δB_w the amplitude of waves having wave length $O(r_g)$, we find that $\kappa_{\parallel} < 1.5\text{x}10^{28}$ cm^2/sec corresponds to $\delta B_w/B \sim 10^{-3}$ (T GeV)$^{1/2}$ and hence $\delta B_w/B$ should range between 10^{-3} and 1 for 1-10^6 GeV protons.

The condition (9.1) is stronger than the requirement that the shock should appear approximately plane (namely $\kappa_1/V_1 < R_s$ or $\kappa_1 < R_s\dot{R}_s$ for isotropic diffusion). However, if the diffusion coefficient is highly anisotropic (as it probably is for $T \lesssim 10^6$ GeV/nuc), this requirement is always satisfied. Furthermore, it should be noted that κ_1 refers to the component of the diffusion coefficient normal to the shock and it could therefore be identified with $\kappa_{\parallel}\cos^2\theta = \kappa_{\parallel}/3$ if an average is taken over the (spherical) shock surface. Thus we require that $\kappa_{\parallel} \lesssim 1.5\text{x}10^{28}$ cm^2/sec.

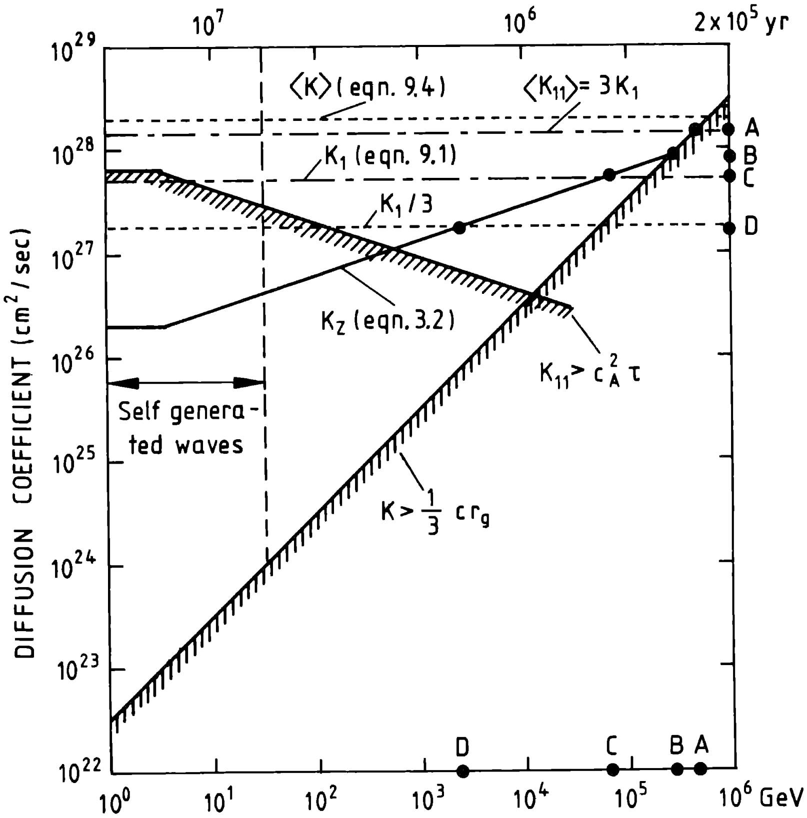


Fig. 12. Diffusion coefficients versus energy/nucleon and confinement time. The maximum diffusion coefficient which would allow the equilibrium spectrum to be achieved is κ_1 (equation (9.1)); this intersects the diffusion coefficient for escape from the galaxy at point C (T $\sim$ 6×10^4 GeV/nuc). If $\langle\kappa_\parallel\rangle = 3\kappa_1$ is used, the intersection point yields an upper limit to acceleration at T $\sim 10^6$ GeV/nuc. Allowance for the energy dependence of the diffusion coefficient (equation (6.7)) yields an upper limit T $\sim 2\times10^3$ GeV/nuc (point D). The average diffusion coefficient κ, resulting from an equilibrium between shock amplification and non-linear damping (equation (9.4)), is slightly greater than $3\kappa_1$. The diffusion coefficient has a lower limit $\kappa = cr_g/3$ which yields an upper limit, using $\langle\kappa_\parallel\rangle$, of T $\sim 5\times10^5$ GeV/nuc. Second order Fermi acceleration should not be important if the cosmic ray spectrum is to be determined by shock acceleration ($\kappa_\parallel > c_A^2\tau$); this could give rise to difficulties if the diffusion coefficient is as small as $\kappa_1/3$ or κ_z. Self-generated waves occurring near the shock can permit acceleration up to 10-50 GeV/nuc so that the above arguments should be ignored in this region of the diagram (185). It is evident that κ_1 or $\langle\kappa_\parallel\rangle$, as shown, would be acceptable but κ_z is too small in the range T $\leq 10^3$ GeV/nuc. [Based on arguments presented by Ginzburg and Ptuskin (67), Cesarsky and Lagage (68) and Krimsky and Petukhov (183).]

These constraints on κ_1 and $\kappa_{\parallel}$ are displayed in Figure 12 together with $\kappa_z(T)$, representing the coefficient for diffusion perpendicular to the plane of the galaxy (3.2). It would seem that there is no real difficulty in satisfying the requirements for shock acceleration by supernova shocks until energies of κ order of $5x10^5$-10^6 GeV/nuc are reached. However it should be noted that the form for τ_a used in obtaining (9.1) is rather crude and if we were to allow for the fact that the shocks are not infinitely strong (e.g. $V_1/V_2 \sim 3.3$) and the diffusion coefficient depends on energy (e.g. $\kappa_1 \propto T^{1/3}$ as in (3.2)) then a further factor ~ 3 is required (for acceleration from an injected population at very low energies) which corresponds to reducing the upper limit on the energy achievable by a factor ~ 27 to $2x10^3$-$4x10^4$ GeV/nuc. The essential problem here is that, apart from some conceptual difficulties, we do not know what the typical values of $\kappa_{\parallel}$ might be in the HISM. The arguments given above concerning the anisotropy of the diffusion coefficient could equally well apply to κ_z, but on the other hand our estimate of the latter is entirely dependent on the most elementary leaky box model with $L \sim 200$ pc, $\tau_0 = 2x10^7$ years and it is assumed that the diffusion coefficient so obtained is somehow representative of $\kappa_{\parallel}$, which is of interest to us. In fact, the escape of cosmic rays from the galaxy may well be largely controlled by convective transport associated with the supersonic turbulence in the HISM (12,73) and by a galactic wind, so that the above difficulty, which has been emphasized by Ginzburg and Ptushkin (67), may not be real.

It has been pointed out that there is a further important constraint on the diffusion coefficient at high energies, namely the equivalent scattering mean free path should not be smaller than the particle gyro-radius so that $\kappa > cr_g/3$ (67,68,18). This constraint is also shown in Figure 12, where we see that it yields an upper limit on the energy of ~ 3-$5x10^5$ GeV for protons and ~ 1-$3x10^5$ GeV/nuc for other species if $B_g \sim 1$ μGauss. To reach these energies as a result of acceleration by a single shock, the magnetic field must be disordered on a length scale of 0.1-1 pc - which is perhaps not unreasonable. With the value of B_g assumed, the upper limit quoted is not quite as severe as that obtained from considerations of κ_z, however if B_g is as small as 10^{-7} Gauss, as argued by Hall (72,73), the upper limit to the energy achievable would again be reduced to $\sim 10^4$ GeV/nuc.

An additional constraint has been pointed out by Ginzburg and Ptuskin (67), namely that second order Fermi acceleration should not be important if the cosmic ray energy spectrum is to be determined by shock acceleration alone. For ultra-relativistic particles the time scale for second order Fermi acceleration by the Alfvén waves which cause pitch angle scattering is $\kappa_{\parallel}/V_A^2$ (184) which is in general much greater than the shock acceleration time $4\kappa_1/V_s^2$ since $V_s >> V_A$. However, second oder Fermi acceleration can operate over the whole period of containment in the galaxy, so the essential constraint is $\kappa_{\parallel}/c_A^2 > \tau = \tau_0(T \text{ GeV/nuc})^{-1/3}$, again a lower limit on $\kappa_{\parallel}$. This does not appear to be a serious constraint, however, since it approaches the upper limit required for shock acceleration only at energies less than ~ 1 GeV/nuc where there is no requirement that shock acceleration be dominant.

At low energies, the magnetic field irregularities required for scattering can be locally generated by cosmic ray streaming instabilities associated with the shock itself (4). According to Völk (185) however, if non-linear damping of the unstable waves is taken into account, the upper limit on the energy to which particles can be accelerated relying only on self-generated waves is only 10-50 GeV/nuc (see Figure 12). Since linear effects strongly damp all magnetohydrodynamic waves other than Alfvên waves propagating parallel to the magnetic field and the latter are eventually damped by non-linear effects (186,187) it is necessary to examine carefully the nature of the magnetic irregularities required for particle scattering.

In view of the evidently violent nature of the HISM it seems difficult to imagine that the magnetic field is completely smooth on scales of the order of 0.1-1 pc. Indeed we see from Figure 10 that weak shock waves can be expected to pass any point in space with a frequency corresponding to such length scales. Weak shocks are not in themselves likely to be very effective in cosmic ray scattering since the large scale magnetic field changes they produce are relatively small, however they are capable of amplifying existing Alfvên (and other) waves (188) and generating new waves as a result of the cosmic ray streaming they induce (70,71). It is possible that the magnetic field in the HISM is more or less random on length scales of the order of 1 pc, which would satisfy the requirement that $\kappa_{\parallel} \sim 1.5\times10^{28}$ cm^2/sec for $T \sim 10^6$ GeV. If this turbulence could cascade to smaller wave lengths maintaining a Kolmogrov spectrum then the diffusion coefficient would be easily small enough for our purpose for energies less than 10^6 GeV and it would have the energy dependence required to account for the apparent escape lifetimes (189). It is not clear however that the necessary cascading will take place at a sufficient rate to overcome the effects of wave damping which become progressively more important at small wave lengths (187,188).

For the case of non-linear Landau damping of Alfvên waves (186):

$$\frac{d}{dT}(\delta B_w^2) = -\delta B_w^4/B^2\tau_1 \;, \quad \tau_1 = \sqrt{(8/\pi)}/c_g\langle k\rangle \;, \qquad (9.2)$$

where $\langle k\rangle$ is the wave number corresponding to the particle momentum of interest. With values of the quantities involved which are appropriate to the HISM we find that

$$\delta B_w^2/\delta B_{wo}^2 = 1/\left[1 + (\delta B_{wo}^2/B^2)(t/\tau_1)\right] \;, \qquad (9.3)$$

where δB_{wo} is the wave amplitude at t = 0. Consequently,

$$\langle\kappa\rangle = \frac{1}{3}\, cr_g \left\langle (B^2/\delta B_{wo}^2) + (t/\tau_1)\right\rangle \sim 2\times10^{28}\, t_4 \;cm^2/sec \;. \qquad (9.4)$$

Thus if the waves are regenerated at intervals of the order of 10^4 years, an adequate diffusion coefficient can be maintained on the average. It is important to note that in the case of non-linear damping, the temporal behaviour is not an exponential decay, but inverse time; hence the decay rate becomes progressively longer at later times and is not sensitive to the initial value of δB_w, as long as $\delta B_{wo} >> \delta B_w$. A more detailed analysis of these points is at present being carried out (188).

Bell's suggestion that Alfvên wave generation occurs as a result of instabilities induced by cosmic ray streaming ahead of shock fronts has been criticised by Morrison et al. (189) on the grounds that, with the effects of cosmic ray pressure included in the dispersion equation, the phase speeds of the most unstable waves exceed the Alfvên speed by a very large factor and may even be "supersonic". These authors quote an approximate result for the phase speed of the most rapidly growing wave:

$$v_{ph} = c_A \left[1 + \frac{\pi}{5} (E_*/m)(kc/\Omega_*)^{1/2}(\epsilon\xi/c_A^2)\right]^{1/2} , \qquad (9.5)$$

where the cosmic rays are supposed to have a power law spectrum above a cut-off energy E_*, Ω_* is the corresponding gyrofrequency, ϵ is the ratio of the cosmic ray to plasma number density and ξ the anisotropy. Taking $E_* = 10^{-3}$ ergs, $\epsilon = 10^{-7}$, $kc/\Omega_* = 1$, $\xi \leq 1$ and $c_A = 35$ km/sec, we obtain $v_{ph} = 2.4\ c_A$. This is very much less than the minimum shock speed in the HISM, namely $c_s \sim 200$ km/sec; hence the cosmic rays do not stream supersonically and can therefore be accelerated as described in section 5. If we take $c_A = 110$ km/sec corresponding to $B_g = 3\ \mu$Gauss, we obtain $v_{ph} = 1.3\ c_A$ which is again substantially less than the sound speed in this case ($c_s = 230$ km/sec). If $B_g = 10^{-7}$ Gauss, then $v_{ph} \sim$ 100 km/sec which is much greater than c_A, but smaller than the sound speed $c_s = 200$ km/sec. [The formal analysis of Morrison et al. does not differ significantly in substance from that of Zweibel (190) whose conclusions are essentially those presented here.] It should be noted that the waves generated by streaming instabilities propagate in the direction of the magnetic field, which is not necessarily normal to the shock: in this case the relative speed of the scattering medium is $V_1 = V_s - v_{ph}\cos\theta$. It is certainly the case that such a reduction in relative speed can reduce the effectiveness of shock acceleration (10) but it does not appear to be sufficient to prevent it from occurring.

A different approach to the problem of generating the magnetic field irregularities required to scatter cosmic rays has been taken by Hall (72,73). He considers the effect of large scale supersonic turbulence in creating pressure anisotropies which cause firehose and mirror instabilities which in turn produce small scale magnetic irregularities capable of scattering cosmic rays. Using observations of electron density fluctuations in the galaxy obtained from pulsar scintillation data, he estimates the plasma pressure anisotropies and the associated hydromagnetic wave field which might exist. The latter is then used to determine a cosmic ray diffusion coefficient which is in reasonable agreement with the shape of the cosmic ray path length distribution apart from a scale factor which has not been adequately determined. In fact this work is complementary to the other approaches to the production of magnetic field irregularities we have outlined: it would be interesting to have the effects of finite cosmic ray pressure and pressure anisotropies included in the analysis, together with an investigation of the effects of linear and non-linear damping on the wave spectrum.

10. Non-linear Cosmic Ray Shocks

It has been noted at several points in this review that the linear or "test particle" approach which we have so far applied to the problem of cosmic ray interaction with shocks encounters certain difficulties. For example, it is evident from Figure 5 that if $\gamma_c = 4/3$ the cosmic ray pressure behind a strong shock may become very large if the compression ratio approaches 4; hence, given the fact that $p_{c1} \sim p_g$ in the HISM, the cosmic ray pressure produced by the shock may be the dominant pressure in the medium. This contradiction is also apparent in Figure 10, where the cosmic ray pressure in supernova remnants, which are of most interest for cosmic ray acceleration, is shown to be comparable with the gas pressure so that the cosmic rays must play a role in the dynamics of the expanding remnants. In connection with the problem of the diffusion coefficient discussed in section 9, it is evident that there is likely to be an intimate connection between the cosmic rays and the diffusion coefficient since cosmic ray anisotropies can produce the hydromagnetic waves which in turn scatter the particles and tend to quench the anisotropies which cause them.

These points and others which are perhaps more subtle, are all indications that the problem of cosmic ray acceleration by shocks is far more complicated than implied by the discussion given in the previous sections. That is, we cannot expect in general to be given the diffusion coefficient κ and the velocity field in the medium $\underline{V}$ and be satisfied to calculate the behaviour of cosmic rays using linear transport equations as in sections 5 and 6, for example. In a realistic treatment it is necessary instead that we simultaneously determine the nature of the wave field (and hence the diffusion coefficient) and the reaction of the medium to cosmic ray and wave pressure gradients. Furthermore, we must consider the problem of cosmic ray injection, which opens a Pandora's box of questions concerning the state of the background plasma, the nature of collisionless shocks, pre-acceleration in plasma turbulence and so on (13,191-194). Although the literature concerned with the non-linear problem is not very extensive, the concepts and calculations involved are rather complicated and there is no space to give a detailed review here. Instead, we describe the general outline of the problem using Figure 13 as a basis, and provide a list of references.

In Figure 13, we show the system divided into three components, namely the plasma, the cosmic rays and the hydromagnetic wave field. The plasma may be described in terms of its distribution function, which is important in considering the problem of cosmic ray injection, and also in terms of hydrodynamic quantities such as pressure, density, velocity (p_g, ρ, $\underline{V}$) and the mean magnetic field ($\underline{B}$). The cosmic rays can be described in terms of their distribution function (U, S) or in terms of the corresponding "hydrodynamic" quantities, namely the cosmic ray pressure and energy flux (p_c, F_c). The wave field can likewise be described in terms of its power spectrum as a function of wave number, or, where appropriate, the integral quantity δB_w^2, the mean square magnetic field fluctuation. The wave field determines the cosmic ray diffusion coefficient and, as a consequence, the nature of the coupling between the cosmic rays and the background medium.

The "linear" approach which we have reviewed in the previous sections, assumes that the quantities in the plasma and wave boxes of

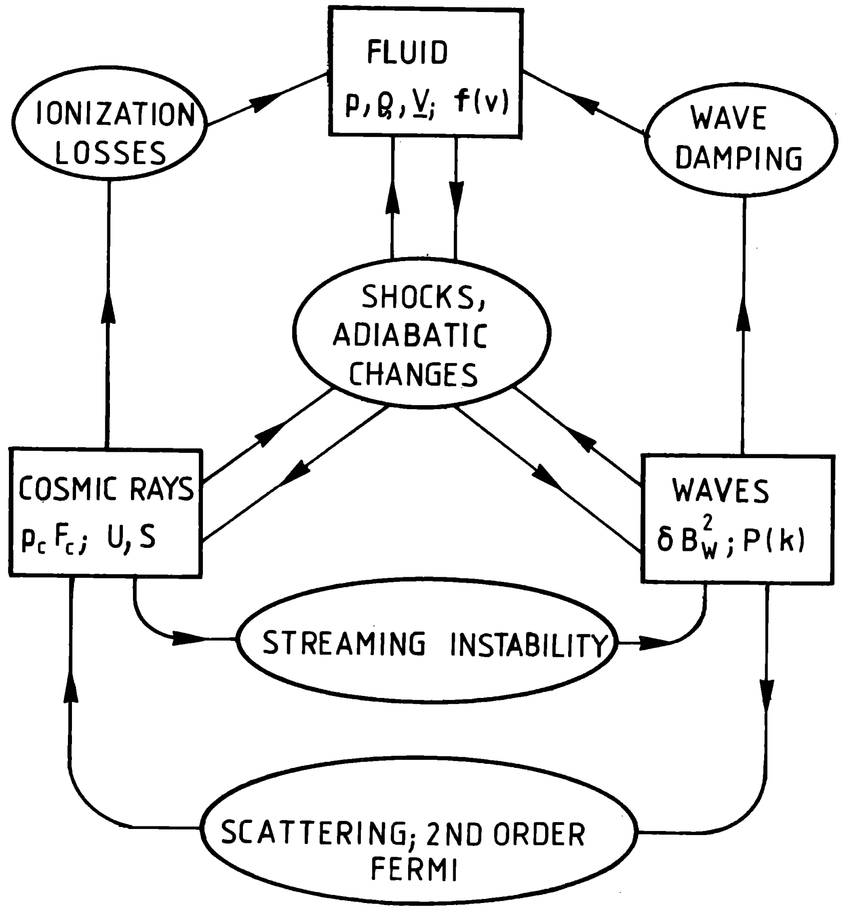


Fig. 13. A diagram showing schematically the non-linear interaction scheme described in the text. The arrows represent the direction of energy flow.

Figure 13 are determined independently and that the cosmic rays simply respond to the resulting diffusion coefficient and velocity field. We may arbitrarily assume a certain rate of injection of cosmic rays from the background plasma and also take into account cosmic ray energy change due to ionization losses, etc.

Ignoring the existence of cosmic rays to begin with, it is also possible to consider the interaction of waves with a given plasma velocity field, including shocks. It is found that the wave field must also gain and lose energy as a result of plasma compressions and rarefractions and, in particular, the waves can be strongly amplified by interacting with shocks (195,196). The effects of damping, which feed energy from the wave field back to the plasma may also be considered, as well as the direct effects of wave pressure and energy flux on the dynamics of the plasma as a whole (197-199).

A second non-linear combination can be considered in which the behaviour of the plasma as a whole is prescribed but the cosmic rays and wave field interact in such a manner that the waves which determine the diffusion coefficient are produced by cosmic ray streaming (4,69-71). The waves damp by giving energy to the cosmic rays, both directly and as a consequence of second order Fermi acceleration, and also by giving their energy up to the fluid as a result of various linear and non-linear plasma damping mechanisms.

The third non-linear combination of two of the components shown in Figure 13 comprises the cosmic rays and the plasma, with the wave field (and thus the diffusion coefficient) being given (3,12,13,191-194,200-205). The most straightforward procedure in this case is to treat the hydrodynamic aspects of the interaction first, describing the cosmic rays in terms of their pressure and energy flux and using a suitably defined average diffusion coefficient ($\bar{\kappa}$). Thus, for steady, one-dimensional flow:

$$\rho V = \text{constant} \; , \tag{10.1}$$

$$p_g + p_c + \rho V^2 = \text{constant} \; , \tag{10.2}$$

$$\rho V \left(\frac{1}{2} V^2 + \frac{\gamma}{\gamma - 1} p_g/\rho\right) + F_c = \text{constant} \; , \tag{10.3}$$

representing conservation of mass, momentum and energy respectively. The first moments of (5.1) and (5.2), integrated over T, yield:

$$\frac{dF_c}{dx} = V \frac{dp_c}{du} + \overline{Q}_1 \quad , \tag{10.4}$$

$$F_c = \frac{\gamma_c}{\gamma_c - 1} p_c V - \frac{\bar{\kappa}}{\gamma_c - 1} \frac{dp_c}{dx} \quad , \tag{10.5}$$

where $\overline{Q}_1 = \int_0^\infty TQdT$ can be neglected if cosmic ray injection occurs at very low energies. These equations can be integrated for given $\bar{\kappa}$ to yield shock-like transitions in which the total Mach number $M = V/c_s$ changes from supersonic to subsonic. The flow speed varies smoothly but an internal plasma shock occurs in some circumstances (3,201-203).

The properties of these transitions are such that for sufficiently large values of the upstream pressure ratio ($p_{c1}/p_1 \gtrsim 4.6$ for $\gamma_c = 4/3$) the transitions are always smooth, with cosmic ray diffusion playing a role similar to heat conduction in an ordinary shock (12,13,200). For lower initial pressure ratios, an internal plasma shock must be introduced into the flow if the initial Mach number lies in a certain range but is otherwise also smooth (200-202). In the particular case where $p_g = 0$, smooth solutions are always the rule and the transition has the simple form:

$$V = \frac{1}{2}(V_1+V_2) - \frac{1}{2}(V_1-V_2) \text{ Tanh } x/L \quad , \tag{10.6a}$$

$$V_2/V_1 = (\gamma_c-1)/(\gamma_c+1) + 2/(\gamma_c+1)M_1^2 \quad , \tag{10.6b}$$

where $L = 2\bar{\kappa}/V_1(1-1/M_1^2)$ and $M_1^2 = \rho_1 V_1^2/\gamma_c p_{c1}$. This solution (which is analogous to that obtained for the structure of a shock wave dominated by heat conduction rather than viscosity) shows the usual feature of being narrow if $M_1^2 >> 1$ and broad when $M_1^2 \to 1$. The overall compression ratio $V_1/V_2 \to (\gamma_c+1)/(\gamma_c-1) \sim 7$ as $M_1^2 \to \infty$; in this case most of the kinetic energy of the fluid is converted to cosmic ray energy and the conversion efficiency approaches 98% as required to account for cosmic ray acceleration by shock acceleration in supernova remnants.

In order to complete the solution for this type of non-linear interaction it is necessary to determine the corresponding behaviour of the cosmic ray spectrum (i.e. U and S). That is, we must solve (5.1) and (5.2) with the flow speed determined from (10.1-10.5). This is in general difficult but a solution has been found in one case, where the flow speed is given by (10.6a) and the cosmic ray spectrum far upstream from the transition is a δ-function (203,204). The downstream solution in this case is such that the peak of the spectrum shifts in a manner appropriate to adiabatic compression by a factor V_1/V_2 and a high energy tail develops such that $F_o(p) \propto (p/p_1)^{-\nu}$ where

$$\nu = \frac{3V_1}{V_1-V_2}\left[1 + \frac{2}{\gamma_c+1}\frac{V_2}{V_1-V_2}\right] . \tag{10.7}$$

This index is larger than would be obtained for the same compression ratio in an unmodified shock (cf. 5.10), however this is not surprising since compression ratios larger than 4 are possible in this case and the cosmic ray pressure must remain finite (see Figure 5b). In fact, if $M_1^2 >> 1$, $V_1/V_2 \to 7$ and $\nu \to 7/2(1 + (6/7)(1/6)) = 4$ which is to be compared with $\lambda_1 \to 4$ as $V_1/V_2 \to 4$; of course $p_{c2} \to \infty$ in either case.

On the basis of these results it appears that shocks associated with supernova remnants can be very efficient in accelerating cosmic rays. The transitions may be smooth in the very early and late phases, but in general they contain an internal plasma shock which converts the remaining flow kinetic energy largely into plasma internal energy. It is not certain that smooth transitions really occur at early phases since no dissipation of waves due to streaming instabilities has been included and this could easily change the above result. The cosmic ray spectrum at high energies should always be a power law but the index is not necessarily that given by the linear theory, as suggested by the above example. Finally, the cosmic rays may in some circumstances take up much of the upstream kinetic energy available to the shock. This does not mean that exceptional cosmic ray pressures are produced, rather the opposite, since the cosmic rays can at most absorb the energy available whereas in the linear approach (Figure 5b) there is no limitation on the energy given to cosmic rays. This is probably the resolution of the difficulty implied by Figure 10 which suggests that the long term variations of the cosmic ray intensity at the Sun should be much greater than is observed (112,113,206).

In the above approach to the non-linear problem in which the diffusion coefficient is assumed known, it is necessary that a mean diffusion coefficient $\bar{\kappa}$ be given. Provided it is eventually possible to solve for the cosmic ray spectrum, which determines the weighting involved in $\bar{\kappa}$, this poses no real difficulty since $\bar{\kappa}$ can be merged with x and only results in a distortion of the spatial form of the transition. However, as far as the connection between cosmic rays and plasma is concerned (i.e. injection), the approach says nothing (200). An additional theory must be constructed to determine how the cosmic ray and plasma distribution functions merge, that is, to decide how much of the cosmic ray pressure is to be provided by the acceleration of existing particles or by the injection of fresh particles from the background. We are inclined to the view that this is perhaps the best approach since the physics of the injection process may be very complicated, especially if it involves the strong plasma turbulence associated with a collisionless shock (12,13). On the other hand, it is interesting to consider a different approach where the details of the whole particle distribution function are kept in view as far as possible and the problem not split into two steps as described above. The difficulty with this procedure is that the equation describing the distribution function is not easy to handle and a set of integral-differential equations results if hydrodynamic equations are accepted wherever it is reasonable to do so. Eichler (191) has obtained an approximate solution which represents some progress in this direction but it will almost certainly be necessary to fall back on a numerical treatment (193). An exact solution assuming that the distribution function satisfies a Boltzman equation with a constant effective diffusion coefficient for all particles, has been obtained by Krimsky (194). This solution demonstrates that high energy particles appear quite naturally out of the background plasma even when the transition is completely smooth. A perturbation approach using the linear or test particle solution as a basis has been attempted by Blandford (208) but the significance of the result is obscured by the fact that the corresponding non-linear solution is not unique (200-202).

Although the results that have been obtained for the partially non-linear configurations are interesting and instructive as far as the

central problem of cosmic ray acceleration is concerned, it is difficult to avoid the conclusion that a completely non-linear treatment is necessary. That is, the entire complex represented in Figure 13 must be considered without making too drastic assumptions about any of its components. So far, it has been possible only to write down the relevant set of equations (which is no easy matter in itself) and to obtain a few rather specialised and simplified solutions (70,71,209). Very probably, it will eventually be necessary to resort to numerical methods. However, we expect that the following features will emerge:
1) strong hydromagnetic turbulence will develop throughout the transition;
2) the shock will be smoothed out by cosmic ray pressure gradients;
3) much of the fluid kinetic energy will be transferred to cosmic rays;
4) a power law spectrum at high energies will result for plane, steady transitions without losses;
5) damping of the hydromagnetic turbulence will lead to energy transfer from the cosmic rays to the plasma so that the plasma may be efficiently heated even in the absence of an internal collisionless shock.
The injection problem may eventually have to be treated separately but it is to be expected that 1) a transition of finite width will tend to favour acceleration of particles with large diffusion coefficients (191, 192) and 2) the turbulence generated in the transition, whatever form it takes, will inevitably cause selective heating of the plasma so that ions will be injected in a manner which depends on their current mass-to-charge ratio (12,13). The latter may be the explanation for the observation that the relative elemental and isotopic source abundances of galactic cosmic rays and of the energetic particles produced in large solar flares are very similar (209), if both are the result of shock acceleration in media having "coronal" temperatures of the order of 10^6 K.

11. Conclusions

Although the possibility of energetic particle acceleration in shock waves has been investigated by numerous authors for more than twenty years it is only comparatively recently that its more attractive features, as far as the acceleration of cosmic rays is concerned, have been generally realized. It must be accepted that the idea is not without its difficulties and perhaps unrecognized subtle features, but it is surprising how much progress has been made. Whether or not all the theoretical and conceptual problems encountered so far can be resolved is not clear but one can feel reasonably confident that the theory will not be easy to reject.

In our view, the immediate needs of the theory are: 1) to understand the nature of hydromagnetic turbulence in the HISM as a whole; 2) to understand better the nature of fully non-linear cosmic ray shock structures; 3) to investigate the expansion of supernova remnants without neglecting the cosmic rays as one of the most important components; and 4) to understand better the nature of the galactic wind, its energy sources and in particular the role played in its formation by cosmic rays and extended supernova remnants. These are all very difficult problems and one should not expect rapid progress. In the meantime one is free to speculate about the broader implications of shock acceleration in the universe.

First we note that shock waves must be prevalent in the intergalactic medium, not only in obvious situations such as strong radio galaxies, but also in clusters of relatively normal galaxies. Shock waves originating in supernovae for example, can speed up and become strong when they encounter the dilute cluster medium and there is no reason to suppose therefore that there is not rather efficient acceleration of an extragalactic cosmic ray population in circumstances in which the losses due to universal expansion and other effects are minimized (12,13).

Secondly, shock acceleration must be considered as one of two distinct processes which give rise to solar energetic particles (the other process being magnetic field reconnection, especially at flare sites) (3,211,212). There is no lack of hydromagnetic turbulence in the solar corona and the shocks produced by flares can at times be extraordinarily strong (213). Except in the open magnetic field regions associated with coronal holes, solar flare produced shock waves must propagate for considerable distances through magnetically closed regions which could give rise to very efficient shock acceleration. These facts, together with the observation that the charge states of solar energetic particles often correspond to normal coronal and transition region temperatures (214) in contrast to very high temperatures observed in the vicinity of flares, is evidence in favour of shock acceleration. One would expect however that the gamma-ray emission associated with flares is the result of particles produced at the flare site, perhaps by magnetic reconnection, impinging on the lower atmosphere. It is interesting to note that, at least near the Sun, flare shock waves are likely to be highly non-linear in the sense discussed here.

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