Particle Accelerators inside Spinning Black Holes

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On the basis of the Kerr metric as a model for a spinning black hole accreting test particles from rest at infinity, I show that the center-of-mass energy for a pair of colliding particles is generically divergent at the inner horizon. This shows that not only are classical black holes internally unstable, but also that Planck-scale physics is a characteristic feature within black holes at scales much larger that the Planck length.

Recently, Bañados, Silk and West [2] (BSW) suggested that rotating black holes could serve as particle colliders with arbitrarily high center-of-mass energies. This suggestion was soon criticized. Berti et al [3] pointed out that the BSW mechanism requires fine tuning (a degenerate black hole and a critical angular momentum for one of the particles). Further, they pointed out that in the real world one would obtain only modest center-of-mass energies due to the Thorne limit [4]. Moreover, as they showed in some detail, the effects of gravitational radiation are not ignorable. At about the same time, Jacobson and Sotiriou [5] carefully analyzed the fine tuning required by the BSW mechanism, also pointed out the consequences of the Thorne limit, and showed how the redshift further lowers realizable energies.

Here I show that spinning black holes do catalyze hyper-relativistic particle collisions, not about their outer horizons, but rather in the vicinity of their inner horizons. Moreover, I show that this divergence is a generic feature of black holes in that the result requires no fine tuning at all. This instability is reminiscent of but distinct from the well known Poisson-Israel instability [6].

As shown in [2] and [5], for a pair of particles of mass m that fall from rest at infinity, the center-of-mass energy in the Kerr metric is given by

$$\left(E_{\rm cm}^{^{Kerr}}\right)^2 = \frac{2\,m^2 N}{r(r^2-2r+a^2)} \tag{1}$$

where

$$N = 2a^2(1+r) - 2a(l_1+l_2) - l_1 l_2 (r-2) + 2(r-1)r^2$$
$$-\sqrt{(2(a-l_1)^2 - l_1^2 r + 2r^2)(2(a-l_2)^2 - l_2^2 r + 2r^2)}\,, \tag{2}$$

the black hole is given unit mass, the angular momentum per unit mass of the black hole is given by a and the particles have orbital angular momenta of l_1 and l_2. Here we consider black holes in the range $0 < a < 1$.

The horizons are given by $r_{\pm} \equiv 1 \pm \sqrt{1-a^2}$ and here we are concerned only with the inner horizons $r_{_-}$. To prove the general divergence at $r_{_-}$ first note that the denominator of $E_{\rm cm}^{^{Kerr}}$ obviously vanishes there. For the numerator we note that N evaluates to

$$N_{-} = -2a(l_1+l_2) + l_1 l_2 r_{_+} + 4r_{_-}$$
$$-\sqrt{({l_1}^2 r_{_+} + 4(r_{_-} - a l_1))({l_2}^2 r_{_+} + 4(r_{_-} - a l_2))} \tag{3}$$

at $r = r_{_-}$. Whereas the detailed properties of geodesics in the Kerr metric are, let us say, involved; it is adequate for our purposes here to note the following:

- over the range $0 < a < 1$ particles with $-4 < l_1 < 0$ have no turning points
- particles with $A < l_2 < 2$, where

$$A = \frac{2a}{1+\sqrt{(1-a)\,(1+a)}}, \tag{4}$$

have no turning points.

Whereas there is a critical turning point (inflexion in r) at $r = r_{_-}$ for $l_2 = A$ [7], and whereas $N_{-} = 0$ for $l_2 = A$, this is of no consequence here. Over the stated ranges in l_1 and l_2, N_{-} does not evaluate to zero. This is demonstrated in Fig. 1.

Because of the generic nature of the divergence discussed above, a divergence that suffers none of the limitations of the BSW mechanism, it is reasonable to conclude that the use of the Kerr metric and the point-particle geodesic approximation has not given rise to a fictitious result. Given this, the principal conclusion here is that Planck-scale physics is a characteristic feature of black hole interiors at scales much larger than the Planck length.

Acknowledgments

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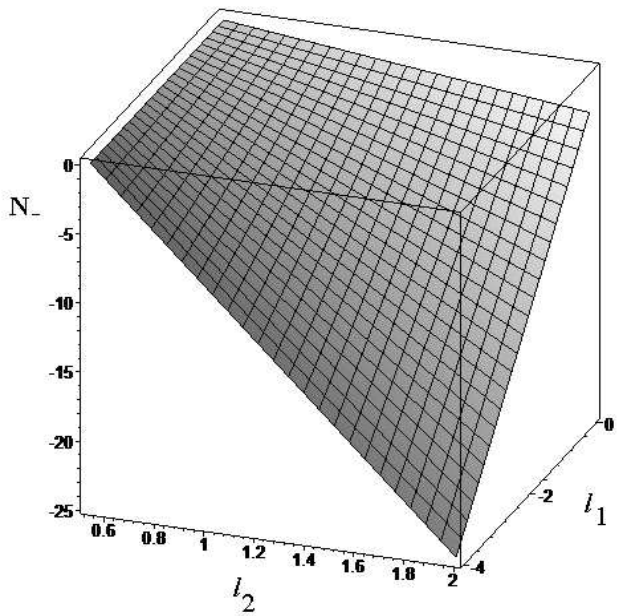

FIG. 1: A plot of N_{-}, given by (3), for $a = 1/2$ within the stated ranges for l_1 and l_2. Other plots in the range $0 < a < 1$ are qualitatively similar.

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[7] The essential content of the BSW mechanism is the choice $a = 1$ and $l_2 = A$.