

# GW170817 Falsifies Dark Matter Emulators

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On August 17, 2017 the LIGO interferometers detected the gravitational wave (GW) signal (GW170817) from the coalescence of binary neutron stars. This signal was also simultaneously seen throughout the electromagnetic (EM) spectrum from radio waves to gamma-rays. We point out that this simultaneous detection of GW and EM signals rules out a class of modified gravity theories, which dispense with the need for dark matter. This simultaneous observation also provides the first ever test of Einstein's Weak Equivalence Principle (WEP) between gravitons and photons. We calculate the Shapiro time delay due to the gravitational potential of the total dark matter distribution along the line of sight (complementary to the calculation in [1]) to be about 1000 days. Using this estimate for the Shapiro delay and from the time difference of 1.7 seconds between the GW signal and gamma-rays, we can constrain violations of WEP using the parameterized post-Newtonian (PPN) parameter  $\gamma$ , and is given by  $|\gamma_{\text{GW}} - \gamma_{\text{EM}}| < 3.9 \times 10^{-8}$ .

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## I. INTRODUCTION

The LIGO-Virgo collaboration detected the inspiral and merger of a binary neutron star (BNS) in the data stream of the LIGO detectors on August 17, 2017 with very high significance (false-alarm-rate of less than 1 per  $8 \times 10^4$  years) and combined SNR of about 32, with the total duration of the detected signal about 100 seconds [2]. A short Gamma-ray Burst (GRB170817A) was detected about 1.7 seconds after this event by the *Fermi* Gamma-ray Burst Monitor [1, 3, 4]. Soon thereafter, an optical transient was detected using the SWOPE telescope (designated as SSS17a/AT2017gfo), enabling a precise measurement of its distance and host galaxy [4, 5]. The position of this transient signal (from the SWOPE observations) is at RA and DEC =  $197.45^\circ$  and  $-23.36^\circ$  respectively [5]. The host galaxy of this merger event is NGC 4993, located at a distance of about 40 Mpc [6]. Subsequently, this signal was also seen in X-rays and radio waves. This is therefore the first GW source for which EM counterparts have been detected. These observations also provide the first direct evidence that the merger of two neutron stars cause a sGRB and also lead to a kilonova powered by radioactive decay of rapid neutron-capture process (r-process) nuclei elements ejected during the explosion [4, 7].

In addition to the above important results, these observations also enable a novel probe of General Relativity (GR) and the equivalence principle for a brand new cosmic messenger, viz. gravitational waves (GWs). The total time it takes for any carrier to reach the Earth from

the Cosmos is equal to the sum of the distance divided by the (vacuum) speed of light and an additional delay due to the non-zero gravitational potential of the cumulative mass distribution along the line of sight.

The latter delay is known as Shapiro delay [8] and has been directly measured for both a static mass distribution as well as a moving mass in the Solar system [9–11]. These solar system measurements have enabled the most precise tests of GR [12]. Shapiro delay is also routinely used as an astrophysical tool to measure the masses of neutron stars in binary pulsars [13–15].

The cumulative Shapiro delay for a cosmic messenger might seem to have only academic interest, both because it can never be measured and because it is much smaller than the vacuum light travel time. However, it does provide an important test of how the various cosmic messengers couple to gravity. The first calculation of this line-of-sight Shapiro delay was done in 1988, following the detection of neutrinos from SN 1987A [16, 17] and the detection of the optical flash about four hours after the neutrino event. It was pointed out, in back-to-back papers [18, 19] (see also [20]), that the neutrinos also encountered a Shapiro delay of about 1-6 months due to the gravitational potential of the intervening matter along the line of sight. We note that this is the only direct evidence to date that neutrinos are affected by GR and obey WEP to a precision of 0.2-0.5%.

Recently, there has been a resurgence of interest in the calculations of line-of-sight Shapiro delay following Wei *et al.* [21] (and citations to it), who pointed out how one can constrain the WEP using simultaneous observations of compact objects throughout the EM spectrum. The violation of WEP in these papers has been quantified in terms of the difference in PPN parameter  $\Delta\gamma$  [12]. Consequently, a wide variety of extragalactic as well as galactic astrophysical objects such as Fast Radio Bursts (FRBs) [22–24], blazars [25], GRBs [21], and pulsars [26–28] have been used to constrain the WEP. A review of

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most of these results can be found in Table 1 of Ref. [24]. For all these papers, a key ingredient is the accurate calculation of galactic as well as extra-galactic Shapiro delay [29]. In some of these works, the total gravitational potential of the Laniakea supercluster of galaxies has been considered [30].

A similar test of the WEP for GWs using line of sight Shapiro delay from a GRB, which simultaneously emits GWs and photons was first proposed by Sivaram [31]. All previous detections by LIGO [32–35] were binary black hole mergers, for which no EM counterparts are expected (See, however [36].) However, the detection of GWs from the first LIGO detection (GW150914) over a frequency range of about 150 Hz within a 0.2 second window, allows us to constrain any frequency-dependent violations of Shapiro delay for GWs. From GW150914 [32], one can constrain frequency dependent violations of the WEP for gravitons to within  $\mathcal{O}(10^{-9})$  [37–39]. Recently, Takahashi [40] has pointed out that for a lensing mass of about  $\sim 1M_{\odot}$ , gravitational waves in the frequency range of ground-based GW detectors do not experience Shapiro delay, because we are in the geometrical optics regime. However, since galactic masses are  $\mathcal{O}(10^{12} M_{\odot})$ , this issue is not of concern to us.

Independently of testing the WEP for different cosmic messengers, the relative Shapiro delay between gravitational waves and photons/neutrinos enables us to test a certain class of modified gravity theories, which dispense with the dark matter paradigm [41] and reproduce Modified Newtonian Dynamics (MOND) [42] like behavior in the non-relativistic limit. Such modified theories of gravity have been dubbed “Dark Matter Emulators”. Some examples of these Dark Matter (DM) emulator theories include Bekenstein’s TeVeS theory [43] and Moffat’s Scalar-Tensor-Vector gravity theory [44]. These models have the property that, in the extreme weak field regime relevant to cosmology, gravitational waves propagate on different geodesics from those followed by photons and neutrinos. Even though the actual model is one of modified gravity, the different geodesics can both be viewed as those of GR, but coupled to different matter sources. The null geodesics of GWs are sourced by only the baryonic matter, however, photons/neutrinos propagate on null geodesics sourced by baryonic matter and the much larger pools of dark matter which would be required if GR were correct. Therefore, the differential Shapiro delay between GWs and photons/neutrinos is due to the gravitational potential of only the dark matter. More details on DM emulator theories and some of their predictions made for externally triggered GW searches [45] during the era of initial and enhanced LIGO can be found in our previous works [46–48]. Now that the first simultaneous observation of GWs and photons has occurred, we can carry out our proposed tests. We note that recently Chesler and Loeb [49] have pointed out that some relativistic generalizations of MOND show non-linear behavior in the weak field regime, which is inconsistent with observations from GW150914. There are also other se-

vere constraints on TeVeS from binary pulsars [50] and large-scale structure (LSS) [51, 52].

The outline of this paper is as follows. We provide a brief summary of the GW and EM follow-up observations of GW170817 in Sect. II. We review the predictions of DM emulator theories and the Shapiro delay calculation in Sect. III. Our limits on the violation of WEP are presented in Sect. IV. We conclude in Sect. V.

## II. GW170817 OBSERVATIONS

We provide a brief recap of the GW and EM followup observations of GW170817. More details can be found in the multi-messenger followup paper [4]. GW170817 was detected by the advanced LIGO and Virgo detectors and the signal was consistent with a binary neutron star coalescence with a merger time at 17th August 2017 12:41:04 UTC [2]. The signal lasted for about 100 seconds in the sweet spot of the sensitivity range of the current GW detectors. A corresponding  $\gamma$ -ray signal was detected by *Fermi* Gamma-Ray Burst Monitor about 1.7 seconds after the merger [3]. The  $\gamma$ -ray signal was also confirmed by the Integral satellite [53].

The first detection of an optical transient was by the One-Meter, Two Hemisphere (1M2H) team, which discovered a 17th magnitude transient in the *i*-band using the SWOPE telescope [5]. They also pinned down the location of the transient (dubbed SSS17a) to  $\alpha$  (J2000) = 13h09m48.085s and  $\delta$  (J2000) =  $-23^{\circ}22'53''.343$  at a projected distance of 10.6” at the center of NGC 4993 at a distance of about 40 Mpc [6]. Many other optical teams subsequently confirmed this transient from UV to IR wavelengths. An X-ray counterpart was detected by the Chandra telescope about 9 days after the merger event. Finally, a radio counterpart was detected by the Very Large Array (VLA) about 16 days after the merger event.

## III. SHAPIRO DELAY CALCULATION

We define a *Dark Matter Emulator* as any modified gravity theory for which:

1. Ordinary matter couples to the metric  $\tilde{g}_{\mu\nu}$  ( $\tilde{g}$  denotes the “disformally transformed metric”) that would be produced by general relativity with dark matter; and
2. Gravitational waves couple to the metric  $g_{\mu\nu}$  produced by general relativity without dark matter.

When a neutron star merger occurs they emit GWs and photons simultaneously. If physics is described by a DM emulator model then GWs will arrive earlier compared to photons (That they arrive earlier derives from the extra potential due to dark matter. It is also required

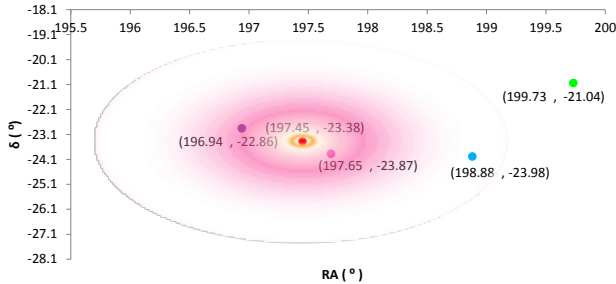


FIG. 1: The angular locations of galaxies which affect the Shapiro delay of any cosmic messenger coming from NGC 4993

to avoid the emission of gravitational Cherenkov radiation [54, 55].) The additional Shapiro delay in the arrival times of photons would be only due to the dark matter

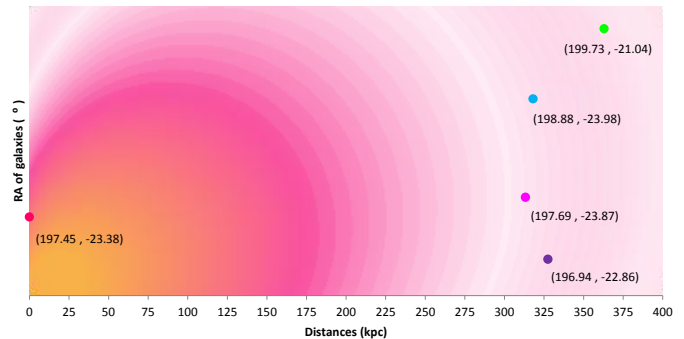


FIG. 2: The distance of the galaxies along the cylindrical line of sight towards NGC 4993

needed if GR was correct. Therefore, one has to include all the contributions coming from the galaxies along the line of sight from NGC 4993 to the Earth.

The main properties of these galaxies are listed in the following Table I.

TABLE I: The properties of the galaxies shown in Fig. 2

Name	RA(°)	δ (°)	Bmag	e_Bmag	PA(°)	BMAG	e_BMAG	Dist (Mpc)	e_Dist (Mpc)
NGC 5068	199.73	-21.04	10.20	0.30		-18.90	0.31	6.60	1.45
NGC 5042	198.88	-23.98	11.81	0.30	19.0	-18.70	0.31	12.65	2.53
ESO 508-011	196.94	-22.86	12.88	0.30	95.0	-19.26	0.31	26.79	5.36
NGC 4993	197.45	-23.38	12.87	0.19	173.2	-20.20	0.20	33.81	5.07
ESO 508-024	197.69	-23.87	12.64	0.30	72.0	-19.98	0.31	33.42	5.01

The columns in Table I are as follows. Bmag: Apparent blue magnitude, e\_Bmag (mag): Error in apparent blue magnitude, PA: Position angle of the galaxy (degrees from north through east), BMAG: Absolute blue magnitude, e\_BMAG (MAG): Error in absolute blue magnitude, Dist (Mpc): Distance (Mpc) and e\_Dist (Mpc): Error in distance (Mpc).

If we consider a cylindrical line of sight, whose radius is 400 kpc from the source to us, then we find using the GLADE catalog (available online at <http://aquarius.elte.hu/glade/>), that there are five galaxies with overlaps inside this cylindrical tube [56]. That means that the photons and the GWs will be affected by these galaxies. If we look at Fig. 2, the closest galaxy to the cylindrical line of sight is 300 kpc away and therefore the total effect of all 5 galaxies inside the tube is small compared to a single galaxy.

At distances as large as 30 Mpc, Shapiro delay will display logarithmic behavior, if one treats the source as

point-like and the metric to be Schwarzschild and is given by:

$$\Delta t_{\text{shapiro}} = (1 + \gamma) \frac{GM}{c^3} \ln \left( \frac{d}{b} \right), \quad (1)$$

where  $\gamma$  is the PPN parameter,  $b$  is the impact parameter, and  $d$  is the distance to the source. For  $M_{\text{MW}} = 10^{12} M_{\odot}$ ,  $d = 400$  kpc,  $b = 8$  kpc, and  $\gamma = 1$  (assuming GR is correct), this equation gives  $\Delta t_{\text{shapiro}}^{\text{MW}} \sim 445$  days [46–48]. Making a more precise estimate of the dark matter contribution using the Navarro-Frenk-White (NFW) profile [57], one gets 300 days. Since the source is now

located at a distance of 40 Mpc and that would give a value of 650 days just due to the Milky Way (MW). The contribution due to NGC 4993 will be of order 300 days and can even be more depending on the location of the merger inside the source galaxy. Finally, due to the additional galaxies in between, there will be an additional delay of order 100 days. Therefore the total Shapiro delay due to the dark matter component would be of the order of 1000 days. In principle, to obtain a more robust estimate on WEP violation, the Shapiro delay due to the baryonic matter needs to be calculated [58], but since the total baryonic mass is negligible compared to the total dark contribution, we do not include its effects. However, the baryonic contribution is not needed for testing DM emulator theories.

The precision of this calculation is not important, only the order of magnitude. Because GR predicts coincident arrival times for photons and gravitational radiation, whereas DM emulators predict delays of over a year, the simultaneous optical detection of GW170817 immediately and decisively falsifies DM emulator models.

We also note that an independent estimate of the Shapiro delay was carried in the joint GW-gamma ray observational paper [1]. In that work, they considered the contribution of the Milky way (for which a Keplerian potential with a mass of  $2.5 \times 10^{11} M_{\odot}$  was assumed) outside a sphere of 100 kpc.

#### IV. CONSTRAINTS ON WEP

Once the Shapiro delay for a given mass distribution is calculated along a line of sight to GW170817, if gravitational waves and photons arrive from the same source within a time interval ( $\Delta t$ ), after traversing 40 Mpc, one can constrain the violations of WEP in terms of the PPN parameter  $\Delta\gamma = |\gamma_{\text{GW}} - \gamma_{\text{EM}}|$  and the calculated Shapiro delay  $\Delta t_{\text{shapiro}}$  [22]:

$$\Delta\gamma \leq 2 \frac{\Delta t}{\Delta t_{\text{shapiro}}} . \quad (2)$$

If we consider the  $\Delta t$  to be time interval between the GRB arrival time (detected by *Fermi*) and merger time detected by the LIGO-Virgo detectors and from our calculated value of  $\Delta t_{\text{shapiro}}$  we obtain  $\Delta t = 1.7$  secs. Using this value of  $\Delta t$ , we get  $\Delta\gamma < 3.9 \times 10^{-8}$ . We note that this limit is more stringent than that obtained in Ref. [1], which obtained  $\Delta\gamma \sim \mathcal{O}(10^{-6} - 10^{-7})$ , because in the latter a more conservative estimate of the Shapiro delay has

been made.

#### V. CONCLUSIONS

The LIGO-Virgo interferometers detected the coalescence of a binary neutron star candidate on 17th August 2017 and this GW event has been dubbed as GW170817A. This is the first GW source for which EM counterparts were also detected throughout the spectrum ranging from  $\gamma$ -rays (about 1.7 seconds later) to optical (less than 11 hours after), X-rays (9 days later), and radio (16 days later) after the GW detection [1, 2, 4]. These multi-wavelength observations have confirmed the basic picture that binary neutron star mergers give rise to short GRBs and a kilonova/macronova caused by r-process nucleosynthesis [4, 7].

Following our previous works [28, 37, 46–48, 58], we calculated the line of sight Shapiro delay from the total dark potential towards GW170817 to be about 1000 days. This calculation is complementary to a similar estimate done in Ref [1], which considered the Milky way contribution and assumed a Keplerian potential for the same. The observations of EM counterparts also allow us to test WEP for photons. From the difference in the arrival times between the  $\gamma$ -rays and GWs, we point out that gravitons propagate on the same null geodesics, thus obeying WEP. The accuracy of WEP can be quantified using the difference in PPN  $\gamma$  parameters between the GWs and photons and is given by  $|\gamma_{\text{GW}} - \gamma_{\text{EM}}| < 3.9 \times 10^{-8}$ .

We also point out that these observations rule out a whole class of modified theories of gravity designed to dispense with the need for dark matter, called dark matter emulators. If these dark matter emulator models were correct, GWs would have arrived about 1000 days earlier than the photons, since the GWs would only experience the gravitational potential due to the visible matter in these theories.

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[1] B. P. Abbott et al. (LIGO Scientific, VIRGO, Fermi Gamma-Ray burst Monitor, INTEGRAL) (2017), 1710.05834.  
 [2] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. **119**, 161101 (2017), 1710.05832.

[3] A. Goldstein et al. (2017), 1710.05446.  
 [4] B. P. Abbott et al. (LIGO Scientific, VIRGO, Transient Robotic Observatory of the South, The 1M2H Team, Virgo, Euro VLBI Team, The VINROUGE, IceCube, GRAVitational Wave Inaf TeAm, CAASTRO s, Pi of the

- Sky, MASTER, The Swift, The CALET, AstroSat Cadmium Zinc Telluride Imager Team, The BOOTES, The Fermi Large Area Telescope, IKI-GW Follow-up, The DLT40, HAWC, AGILE Team, University The Chandra Team at McGill, ALMA, NuSTAR s, LIGO Scientific, the DES, LOFAR, IPN, The Insight-Hxmt, The MAXI Team, ANTARES, KU, The Dark Energy Camera GW-EM, The Pierre Auger, H. E. S. S.), *Astrophys. J.* **848**, L12 (2017), 1710.05833.
- [5] C. D. Kilpatrick et al. (2017), 1710.05434.
- [6] W. L. Freedman, B. F. Madore, B. K. Gibson, L. Ferrarese, D. D. Kelson, S. Sakai, J. R. Mould, R. C. Kennicutt, Jr., H. C. Ford, J. A. Graham, et al., *Astrophys. J.* **553**, 47 (2001), astro-ph/0012376.
- [7] S. J. Smartt et al. (2017), 1710.05841.
- [8] I. I. Shapiro, *Physical Review Letters* **13**, 789 (1964).
- [9] I. I. Shapiro, M. E. Ash, and M. J. Tausner, *Physical Review Letters* **17**, 933 (1966).
- [10] B. Bertotti, L. Iess, and P. Tortora, *Nature (London)* **425**, 374 (2003).
- [11] E. B. Fomalont and S. M. Kopeikin, *Astrophys. J.* **598**, 704 (2003), astro-ph/0302294.
- [12] C. M. Will, *Living Reviews in Relativity* **17**, 4 (2014), 1403.7377.
- [13] J. H. Taylor, Jr., *Reviews of Modern Physics* **66**, 711 (1994).
- [14] P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts, and J. W. T. Hessels, *Nature (London)* **467**, 1081 (2010), 1010.5788.
- [15] A. Corongiu, M. Burgay, A. Possenti, F. Camilo, N. D'Amico, A. G. Lyne, R. N. Manchester, J. M. Sarkissian, M. Bailes, S. Johnston, et al., *Astrophys. J.* **760**, 100 (2012), 1210.1167.
- [16] R. M. Bionta, G. Blewitt, C. B. Bratton, D. Casper, and A. Ciocio, *Physical Review Letters* **58**, 1494 (1987).
- [17] K. Hirata, T. Kajita, M. Koshiba, M. Nakahata, and Y. Oyama, *Physical Review Letters* **58**, 1490 (1987).
- [18] M. J. Longo, *Physical Review Letters* **60**, 173 (1988).
- [19] L. M. Krauss and S. Tremaine, *Physical Review Letters* **60**, 176 (1988).
- [20] J. D. Franson, *New Journal of Physics* **16**, 065008 (2014), 1111.6986.
- [21] H. Gao, X.-F. Wu, and P. Mészáros, *Astrophys. J.* **810**, 121 (2015), 1509.00150.
- [22] J.-J. Wei, H. Gao, X.-F. Wu, and P. Mészáros, *Physical Review Letters* **115**, 261101 (2015), 1512.07670.
- [23] S. J. Tingay and D. L. Kaplan, *Astrophys. J. Lett.* **820**, L31 (2016), 1602.07643.
- [24] X.-F. Wu, J.-J. Wei, M.-X. Lan, H. Gao, Z.-G. Dai, and P. Mészáros, *Phys. Rev. D* **95**, 103004 (2017), 1703.09935.
- [25] J.-J. Wei, J.-S. Wang, H. Gao, and X.-F. Wu, *Astrophys. J. Lett.* **818**, L2 (2016), 1601.04145.
- [26] Y.-P. Yang and B. Zhang, *Phys. Rev. D* **94**, 101501 (2016), 1608.07657.
- [27] Y. Zhang and B. Gong, *Astrophys. J.* **837**, 134 (2017), 1612.00717.
- [28] S. Desai and E. O. Kahya, *ArXiv e-prints* (2016), 1612.02532.
- [29] A. Nusser, *Astrophys. J. Lett.* **821**, L2 (2016), 1601.03636.
- [30] J.-J. Wei, X.-F. Wu, H. Gao, and P. Mészáros, *JCAP* **8**, 031 (2016), 1603.07568.
- [31] C. Sivaram, *Bulletin of the Astronomical Society of India* **27**, 627 (1999).
- [32] B. P. Abbott et al. (Virgo, LIGO Scientific), *Phys. Rev. Lett.* **116**, 061102 (2016), 1602.03837.
- [33] B. P. Abbott et al. (VIRGO, LIGO Scientific), *Phys. Rev. Lett.* **118**, 221101 (2017), 1706.01812.
- [34] B. P. Abbott et al. (Virgo, LIGO Scientific), *Phys. Rev. Lett.* **119**, 141101 (2017), 1709.09660.
- [35] B. P. Abbott et al. (Virgo, LIGO Scientific), *Phys. Rev. Lett.* **116**, 241103 (2016), 1606.04855.
- [36] A. Loeb, *Astrophys. J. Lett.* **819**, L21 (2016), 1602.04735.
- [37] E. O. Kahya and S. Desai, *Phys. Lett.* **B756**, 265 (2016), 1602.04779.
- [38] X.-F. Wu, H. Gao, J.-J. Wei, P. Mészáros, B. Zhang, Z.-G. Dai, S.-N. Zhang, and Z.-H. Zhu, *Phys. Rev. D* **94**, 024061 (2016), 1602.01566.
- [39] M. Liu, Z. Zhao, X. You, J. Lu, and L. Xu, *Physics Letters B* **770**, 8 (2017), 1604.06668.
- [40] R. Takahashi, *Astrophys. J.* **835**, 103 (2017), 1606.00458.
- [41] G. Bertone, D. Hooper, and J. Silk, *Physics Reports* **405**, 279 (2005), hep-ph/0404175.
- [42] B. Famaey and S. S. McGaugh, *Living Reviews in Relativity* **15**, 10 (2012), 1112.3960.
- [43] J. D. Bekenstein, *Phys. Rev. D* **70**, 083509 (2004), astro-ph/0403694.
- [44] J. W. Moffat, *JCAP* **3**, 004 (2006), gr-qc/0506021.
- [45] B. Abbott et al. (VIRGO, LIGO Scientific), *Class. Quant. Grav.* **25**, 114051 (2008), 0802.4320.
- [46] E. O. Kahya, *Classical and Quantum Gravity* **25**, 184008 (2008), 0801.1984.
- [47] S. Desai, E. O. Kahya, and R. P. Woodard, *Phys. Rev. D* **77**, 124041 (2008), 0804.3804.
- [48] E. O. Kahya, *Physics Letters B* **701**, 291 (2011), 1001.0725.
- [49] P. M. Chesler and A. Loeb, *Physical Review Letters* **119**, 031102 (2017), 1704.05116.
- [50] P. C. C. Freire, N. Wex, G. Esposito-Farèse, J. P. W. Verbiest, M. Bailes, B. A. Jacoby, M. Kramer, I. H. Stairs, J. Antoniadis, and G. H. Janssen, *Mon. Not. R. Astron. Soc.* **423**, 3328 (2012), 1205.1450.
- [51] S. Dodelson, *International Journal of Modern Physics D* **20**, 2749 (2011), 1112.1320.
- [52] R. Reyes, R. Mandelbaum, U. Seljak, T. Baldauf, J. E. Gunn, L. Lombriser, and R. E. Smith, *Nature (London)* **464**, 256 (2010), 1003.2185.
- [53] V. Savchenko et al. (2017), 1710.05449.
- [54] C. M. Caves, *Annals of Physics* **125**, 35 (1980).
- [55] G. D. Moore and A. E. Nelson, *Journal of High Energy Physics* **9**, 023 (2001), hep-ph/0106220.
- [56] G. Dalya, Z. Frei, G. Galgoczi, P. Raffai, and R. S. de Souza, *VizieR Online Data Catalog* **7275** (2016).
- [57] J. F. Navarro, C. S. Frenk, and S. D. M. White, *Astrophys. J.* **462**, 563 (1996), astro-ph/9508025.
- [58] S. Desai and E. O. Kahya, *Modern Physics Letters A* **31**, 1650083 (2016), 1510.08228.